: 1 :
RELATIONS AND FUNCTIONS

STUDY NOTES

- **Relation :** A relation R from set X to a set Y is defined as a subset of the cartesian product X × Y. It is written as $R \subseteq \{(x, y) \in X \times Y : xRy\}$
- **Domain :** Let R be a relation from a set X to a set Y. Then the set of all the first components or Coordinate of the ordered pairs belongs to X.

  Domain : of $X = \{x : (x, y) \in R\}$
- **Range :** Let R be a relation from a set X to a set Y. Then the set of all the second components or coordinate of the ordered pair belong to R is called the range of R.

  Types of Relations :
  - ❖ **Empty or Void Relation :** If no element of X to related to any element of X, i.e., $R = \phi \in X \times X$.
  - ❖ **Universal Relation :** If each element of X is related to every element of X, i.e., $R = X \times X$
  - ❖ **Reflexive Relation :** A relation X in a set X is

    (i) reflexive if $(x, x) \in X$ for every $x \in X$.

    (ii) Symmetric if $(x_1, x_2) \in X$ implies that $(x_2, x_1) \in X$ for all $x_1, x_2 \in X$.

    (iii) Transitive, if $(x_1, x_2) \in X$ and $(x_2, x_3) \in X$ implies that $(x_1, x_3) \in X$ for all $x_1, x_2, x_3 \in X$.
  - ❖ **Equivalence Relation :** If set X is reflexive, symmetric and transitive, then it is called an equivalence relation.
- **Function :** Let X and Y be two non-empty sets. A relation from X to Y, i.e., a subset of X × Y is called function.
  - ❖ **One-One or Injective Function :** A function $X \to Y$ is injective if the images of distinct element of X under f are distinct.

    For every $x_1, x_2 \in X$, $f(x_1) = f(x_2)$ implies $x_1 = x_2$.

    [**Note :** Otherwise f has many images called many one.]
  - ❖ **Onto or Surjective Function :** A function $X \to Y$ is said to be onto if every element of Y is the image of some element of X under f. Such that $f(x) = y$.
  - ❖ **Bijective Function :** A function $f : X \to Y$ is said to be one-one and onto, if it is both.
- **Composition of Functions :** Let $f : X \to Y$ and $g : Y \to Z$, be two functions.

  Then the function $gof : X > Z$ is defined by $(gof)(x) = g(f(x))$, for all $x \in X$ is called composition of f and g.
- **Invertible Function :** A function $X \to Y$ is invertible if, $g : Y \to X$. Such that $gof = I_x$ and $fog = I_y$.

  The function g is called inverse function or invertible function. It is denoted by f^{-1}.

  * If $f : X \to Y$ and $g : Y \to Z$ are invertible function.

  Then, $(gof)^{-1} = f^{-1}og^{-1}$.
- **Binary operations :** Let X be a non-empty set. A function $f : X \times X \to X$ is called a binary operation on set X.

  Each ordered pair $(a, b) \in X \times X$ to a unique element $f(a, b)$ in X.

  (i) $a * b = a * a$ for every $a, b \in X$

  (ii) $(a * b) * c = a * (b * c) \forall\ a, b, c \in X$

  (iii) $a * e = a = e * a \quad \forall\ a \in X$

QUESTION BANK

MULTIPLE CHOICE QUESTIONS

1. Let A be a set, such that the relation $I_A = \{(a, a) : a \in A\}$ on A is called :

(a) Empty relation (b) Inverse relation (c) Identity relation (d) Universal relation

2. A function $f : A \to B$ is an onto, if :

(a) range = co-domain (b) range = domain (c) range $\in R$ (d) range $\Rightarrow (0, \infty)$

3. Let a relation X on the set of R is defined as $(a, b) \in X \to 1 + ab > 0$ for all $a, b \in R$.

Then X is :

(a) Reflexive (b) Transitive

(c) Symmetric (d) Reflexive and symmetric

4. Let R be a relation on set N given by : $R = \{(a, b) : a = b - 3, b > 6\}$, then

(a) $(4, 1) \in R$ (b) $(5, 8) \in R$ (c) $(3, 8) \in R$ (d) $(3, 6) \in R$

5. Which of the following is not equivalent relation on integers?

(a) $xRy \Leftrightarrow x = y$ (b) $xRy \Leftrightarrow x < y$

(c) $xRy \Leftrightarrow x - y$ is even (d) $xRy \Leftrightarrow x + y$ is even

6. Let $f : R \to R$; $f(x) = \sin x$ and $g(x) = x^3$. Then which of the following is correct ?

(a) $gof = \sin^2 x$ (b) $gof = \sin x^2$ (c) $gof = \sin^3 x$ (d) $fog = (\sin x)^3$

7. If $f : A \to A$, $g : A \to A$, are two bijections, then which of the following is incorrect?

(a) *fog* is an injection (b) *fog* is an injection (c) *fog* is an invertible (d) *gof* does not exists

8. Let $X = \{1, 2, 3\}$. The number of relations having (1, 2) and (1, 3) are reflexive and symmetric but not transitive is :

(a) 1 (b) 2 (c) 3 (d) 4

9. If a binary operation on R defined by $a * b = a^2 + b^2 + ab$, then $(2 * 3) * 3$ is equal to :

(a) 400 (b) 457 (c) 427 (d) 420

10. If a binary operation on A, such that $A = Q \times Q$ is defind as $(a, b) * (c, d) = (ac, b + ad)$ for $(a, b), (c, d) \in A$. The identity element :

(a) (1, 0) (b) (1, 1) (c) (0, 0) (d) (0, 1)

11. Let R be a relation on $N \times N$, defined as $(a, b)\ R\ (c, d) \Leftrightarrow ad = bc\ \forall\ (a, b)\ (c, d) \in N \times N$

Then, which of the following is correct?

(I) R is reflexive (II) R is symmetric (III) R ts transitive

(a) Only I (b) Both II and III (c) Both I and II (d) All I, II, and III

12. Sharan and Neelam are playing monopoly. The possibilities of getting number on dice is {1, 2, 3, 4, 5, 6}. They make the set of possible out comes as B = {1, 2, 3, 4, 5, 6} and set for these are A = {Sharan, Neelam}.

If $R : B \to B$ defined as $R = \{(x, y) : y$ is divisible by $x\}$ is

(a) Reflexive Only (b) Symmetrically

(c) Transitive and reflexive (d) Equivalence

13. Rekha wants of pair her cloths, she has 3 trousers and 2 shirts. She forms two sets T and S, Like $T = \{T_1, T_2, T_3\}$ and $S = \{S_1, S_2\}$.

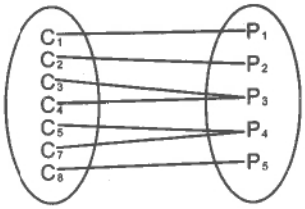
Now she wants is calculate all the possible combinations. How many combinations are possible?

(a) 2^3 (b) 2^6 (c) 2^3 (d) 3^2

14. The Adventure park has a big roller coaster ride. The path traced by this will based on quadratic equation $y = x^2 - 2$. If $f : R \to R$ be defined by $f(x) = x^2 - 2$. Then R is

(a) Bijective (b) Injective (c) Surjective (d) Not Bijective

15. In a mobile phone assume that there are 5 persons in the contact.
If every person in the contact receives a call. Then this function will be

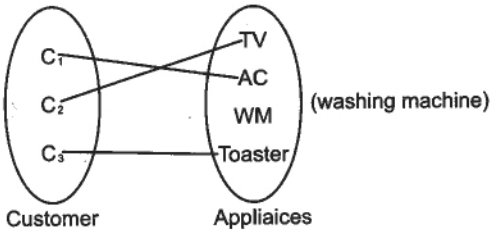


(a) one-one (b) onto
(c) one-one and onto (d) cannot say anything

16. Which of the following express the perimeter P of a square as a function of its area?

(a) $4A = P$ (c) $A^2 = \frac{P^2}{16}$ (b) $A = \frac{P}{4}$ (d) $A = \frac{P^2}{16}$

17. In a home appliance showroom, the product TV, AC, Washing Machine, Toasted are discounted on sale. Three customers are interested and the function is drawn as given. This function is :



(a) one-one (b) onto (c) into (d) bijective

18. Two different teams of boys and girls are formed for the debate competition on gender inequality.
Let $G = \{g_1, g_2, g_3, g_4, g_5\}$ and $B = \{b_1, b_2, b_3, b_4\}$.
Now let $R : G \to G$ and $R = \{x, y : x \text{ and } y \text{ are students}\}$
Then the relation is :

(a) Reflexive only (b) Bijective only (c) Equivalence (d) Reflexive and transitive

19. Let the set $S = \{1, \omega, \omega^2|$ of cube roots of unity. The identity element for multiplication on S is :

(a) 1 (b) ω^2 (c) ω (d) 0

20. Let set A = {children in a family} and relation R is defined on it as aRb if b is brother of a. Then which of the following is correct about the given relation?

(I) Symmetric (II) Reflexive (III) Transitive

(a) Both (I) and (II) (b) Only (III) (c) Only (II) (d) Both (I) and (II)

21. If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{1-x}$, then what is the common domain of $f + g$ and $f - g$?

(a) $0 \le x \le 1$ (b) $0 \le x < 2$ (c) [0, 1] (d) none of these

22. The domain of $\sin^{-1}\left[\log_2\left(\frac{x}{2}\right)\right]$ is :

(a) [1, 4] (b) [0, 4] (c) [–1, 1] (d) [–4, 4]

23. A token is given to all candidates with disabilities and told to come in front when their token number is announced. The function for token numbers is defined as :

$$f(t) = \begin{cases} \frac{t-1}{2}, & \text{when } t \text{ is odd} \\ \frac{-t}{2}, & \text{when } t \text{ is even} \end{cases}$$

Which of the following is true about the given functions?

(a) One-One but not onto
(b) Onto but not one-one
(c) One-One and Onto
(d) Neither One-One nor Onto

24. A function is defined on real numbers such that $f(x + y) = f(x) + f(y)$ for $\forall\ x, y \in R$.

If $f(1) = 5$, then $\sum_{r=1}^{n} f(r)$ is :

(a) $\frac{5n}{2}$ (b) $5n$ (c) $\frac{5n+1}{2}$ (d) $\frac{5n(n+1)}{2}$

25. Let function is defined as $f : (-1, 1) \to X$ such that $f(x) = \tan^{-1}\frac{2x}{1-x^2}$. It is given that the function is both one-one and onto. Which of the following is interval of X?

(a) $\left(0, \frac{\pi}{2}\right)$ (b) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (c) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ (d) $\left(0, \frac{\pi}{2}\right]$

26. Rahul created a dictionary for himself. In which he wrote all the words he found to be difficult. Let W denote the words in his dictionary. The relation is defined as

R : R = {$(x, y) \in W \times W$, the words x and y have atleast one letter common}

Then which of the following is correct about the given relation ?

(a) Symmetric and transitive
(b) Reflexive and symmetric
(c) Reflexive and transitive
(d) Reflexive but not symmetric and transitive

27. A plane is defined as R × R, where R is real. The two subsets of given plane is given as :

M = $\{(x, y) : y = x + 1$ and $0 < x < 2\}$

T = $\{(x, y) : x - y$ is an integer$\}$

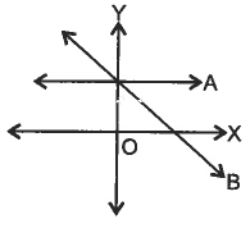
Which of following is correct for the given subsets?

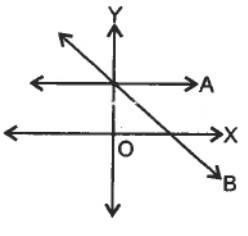
(a) Neither M nor N is equivalence relation on R
(b) Both M and N are equivalence relation on R
(c) M is an equivalence relation on R but not N
(d) N is an equivalence relation on R but not M

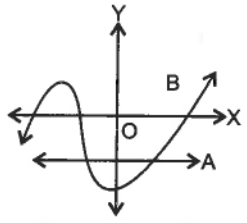
28. Let A and B are finite sets having m and n elements respectively. Find the number of mapping from A to B is:

(a) mn (b) 2^{mn} (c) m^n (d) n^m

29. Which of the following graph represents one-one ?

(I)


(II)


(III)


(a) (I) and (II) (b) (I) and (III) (c) II and III (d) All I, II and III

30. The domain of the given function : $f(y) = \sqrt{y-1} + \sqrt{5-y}$ is :

(a) $[1, \infty)$ (b) $(-\infty, 5)$ (c) $(1, 5)$ (d) $[1, 5]$

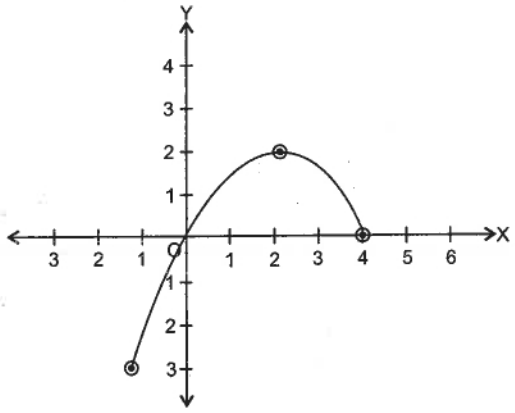
31. If $f(x) = \cos[\pi^2]x + \cos[-\pi^2]x$, where $[x]$ = greatest integer $\le x$, then

(a) $f\left(\frac{\pi}{2}\right) = -1$ (b) $f(\pi) = 1$ (c) $f(-\pi) = -1$ (d) $f\left(\frac{\pi}{4}\right) = 2$

32. The inverse of given function : $f(x) = [1 - (x - 3)^4]^{1/7}$ is :

(a) $3 - (1 - x^7)^{1/4}$ (b) $3 - (1 + x^7)^{1/4}$ (c) $3 + (1 - x^7)^{1/4}$ (d) $3 + (1 + x^7)^{1/4}$

33. Using the graph of the function f. The range of f is :



(a) [–2, 2] (b) [–5, 4] (c) [–3, 2] (d) [2, 2]

34. A function $f(x)$ is defined on real numbers such that

$$f(x) = \begin{cases} x^2 + 2mx - 1 & ; \quad x \le 0 \\ mx - 1 & ; \quad x > 0 \end{cases}$$

In which of the following range the value of m is defined ?

(a) $(-\infty, 0)$ (b) $(-\infty, \infty)$ (c) $(0, \infty)$ (d) $(0, -1)$

35. Two functions $f(x)$ and $g(x)$ are defined as : $f(x) = (x + 1)^2 \ \forall \ x \ge -1$

$g(x)$ = Reflection of $f(x)$ width respect to line $y = x$. Then which of the following is equal to $g(x)$?

(a) $-\sqrt{x} - 1, x \ge 0$ (b) $\sqrt{x+1}, x \ge -1$ (c) $\frac{1}{(x+1)^2}, x \ge -1$ (d) $\sqrt{x} - 1, x \ge 0$

36. If $f(x) = \frac{nx}{x+1}, x \ne -1$, for what value of $f(f(x)) = x$?

(a) $\sqrt{2}$ (b) -1 (c) 2 (d) 1

37. If $f(x) = y = \frac{x+2}{x-1}$, then

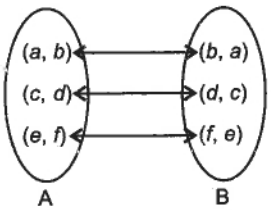
(I) $x = f(y)$ (II) y is increases with x for $x < 1$. (III) f is a rational function of x.

(a) Only I (b) Only II (c) Both I and II (d) Both II and III

38. If $f(x) = \frac{9^x}{9^x + 3}$, then $f\left(\frac{1}{97}\right) + f\left(\frac{2}{97}\right) + \ldots\ldots f\left(\frac{96}{97}\right)$ is equal to

(a) 1 (b) 48 (c) –48 (d) 0

39. From the given diagram of functions what do you say about the function?



(a) Only injective (b) Bijective function

(c) Only surjective (d) None of these

40. A binary operation is given as

$* : R \times R \to R$ defined by $a * b = |a - b|$

$o : R \times R \to R$ defined by $aob = a$

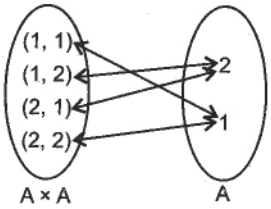
Then which of the following is incorrect

(I) * is commutative (II) o is associative

(III) $a * (boc) = (a * b)\ o\ (a * c)$ (IV) o is commutative

(a) Only II (b) Both (I) and (III) (c) Only IV (d) Both (II) and (IV)

41. Observe the given figure and identify the incorrect statement about the given functions :



(a) 1 is the identity element

(b) * is defined only 1 way

(c) 2 is the inverse of 1

(d) Binary functions $a \times e = 1$

42. The relation R in the set A. Such that set A define all the hotels in a city. The relation is defined as $R = \{(x, y) : x \text{ and } y \text{ have same number of rooms}\}$

Which of the following is correct ?

(I) Symmetric relation (II) Reflexive relation (III) Transitive relation

(a) Only (I) (b) Only (II) (c) Both (II) and (III) (d) All (I), (II) and (III)

43. The relation R is defined in the set A of all the triangles as $R = \{(T_1, T_2) : T_1 \text{ is similar to } T_2\}$ is equivalence. If T_1, T_2 and T_3 are three right triangles with the sides (3, 4, 5), (5, 12, 13) and (6, 8, 10) respectively. Then which triangle among T_1, T_2, and T_3 are related ?

(a) T_1 is related to T_2 (b) T_2 is related to T_3 (c) T_3 is related to T_2 (d) T_3 is related to T_1

44. A relation is defined on a set A of polygons. Such that $R = \{(P_1\ P_2) : P_1 \text{ and } P_2 \text{ have same number fo sides}\}$

What is set of all elements in A related to the right triangle T with sides 3, 4 and 5 ?

(a) set of all polygons (b) set of all triangles

(c) set of all equilateral triangles (d) set of all right triangles

45. Which of the following is incorrect function?

(a) $f : N \to N$ given by $f(x) = x^2$ is injective (b) $f : Z \to Z$ given by $f(x) = x^2$ not injective

(c) $f : R \to R$ given by $f(x) = x^2$ not surjective (d) $f : Z \to Z$ given by $f(x) = x^3$ surjective

46. Signum function is :

(a) one-one (b) onto

(c) both one-one and onto (d) neither one-one nor onto

47. Let $f : R \to R$ is defined as $f(x) = x^{10}$. Which of the following is correct?

(a) f is one-one onto (b) f is many-one onto

(c) f is one-one but onto (d) f is neither one-one nor onto

48. Let * be the binary operation on N. It is given by $a * b$ = LCM of a and b.

Then the identity of $*$ in N is :

(a) 0 (b) a (c) b (d) 1

49. Let R be a reflexive relation on a finite set A lawing 'n' elements and let there be 'm' ordered pairs in R. Then

(a) $m \geq n$ (b) $m \leq n$ (c) $m = n$ (d) $m > 0$

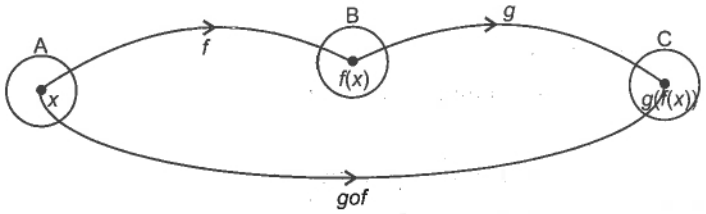
50. A company has four categories of employees given by assistants (A), Clerks (C), Managers (M) and executive officer (E). The salaries provide ₹10,000, ₹25,000, ₹50,000, and ₹1,00,000 to the people who work in the categories A, C, M and E respectively. If $A_1, A_2 \ldots A_5$ are assistants; $C_1, C_2 \ldots C_4$ were clerks; M_1, M_2, M_3 were Managers and E_1, E_2 were executive officers and if R is the relation defined by xRy, where x is the salary to the person y. Express the relation of ordered pair of A_3, C_4, M_2 and E_1

(a) $(10000, A_3)\ (25000, C_4)\ (50000, M_2)\ (100000, E_1)$ (b) $(25000, C_4)\ (10000, A_3)\ (100000, E_1)\ (50000, M_2)$

(c) $(50000, M_2)\ (100000\ A_3)\ (25000\ E_1)\ (10000, M_2)$ (d) $(100000, E_1)\ (50000\ A_3),\ (10000, M_2)\ (25000, C_4)$

INPUT TEXT BASED QUESTIONS

51. Let f: A→B and g : B→C be two functions. Then the composition of f and g, denoted by g of is defined as the function gof ; A→C. $gof(x) = g(f(x))$, $\forall x \in A$



Answer the following questions :

(i) Let f : A → B and g : B → A, such that gof is an identity function on A and fog is an identity is function on B. Then,

(a) $g = f$ (b) $(gof)^{-1} = (fog)^{-1}$ (c) $g = f^{-1}$ (d) $(gof)^{-1} = f^{-1}\, og^{-1}$

(ii) For $f(x) = 2x + 3$ and $g(x) = -x^2 + 1$, the composition function defined by $(fog)x$ is :

(a) $-2x^2 + 5$ (b) $2x^2 + 5$ (c) $x^2 + 5$ (d) $-2x + 5$

(iii) Function f and g are given by, $f(x) = \sqrt{x+2}$ and $g(x) = \text{In}(1 - x^2)$. The domain of composite function $(gof)(x)$ is :

(a) $(-\infty, -1)$ (b) $(0, \infty)$ (c) $(1, \infty)$ (d) $(-\infty, \infty)$

(iv) Function f and g are as set of orders pair $f = \{(-2, 1), (0, 3), (4, 5)\}$; $\{(-1, 1), (3, 3), (7, 9)\}$. The range of function defined by $gof\, x$ is :

(a) $\{-2, 0\}$ (b) $\{-1, 1\}$ (c) $\{1, 3\}$ (d) $\{-3, 3\}$

(v) For $f(x) = \text{In}\, x$, the first derivative of the composite function defined by $F(x) = (fof)x$.

(a) $\frac{1}{x \text{In} x}$ (b) $\text{In}\, x$ (c) $\frac{1}{\text{In} x}$ (d) $x \text{In} x$

ANSWERS

1. (c)	**2.** (a)	**3.** (d)	**4.** (b)	**5.** (b)	**6.** (c)	**7.** (d)	**8.** (a)	**9.** (c)	**10.** (a)
11. (d)	**12.** (c)	**13.** (b)	**14.** (d)	**15.** (b)	**16.** (d)	**17.** (c)	**18.** (c)	**19.** (a)	**20.** (b)
21. (c)	**22.** (a)	**23.** (c)	**24.** (d)	**25.** (b)	**26.** (b)	**27.** (d)	**28.** (d)	**29.** (b)	**30.** (d)
31. (a)	**32.** (c)	**33.** (c)	**34.** (a)	**35.** (d)	**36.** (b)	**37.** (a)	**38.** (b)	**39.** (b)	**40.** (c)
41. (c)	**42.** (d)	**43.** (d)	**44.** (b)	**45.** (d)	**46.** (d)	**47.** (d)	**48.** (d)	**49.** (c)	**50.** (a)

51. (i) (c) (ii) (a) (iii) (a) (iv) (c) (v) (a)

Hints to Some Selected Questions

1. (c) Here, A is called Identity Relation

2. (a) A function f : A → B is an onto, if range = co-domain

4. (b) For the interval (5, 8)

$5 = b - 3$

$b = 8 \qquad \forall\ b > 6$

5. (b) Reflexive : $x \in R$

$x < x$, this is not possible.

6. (c) $gof(x) = g(f(x)) = g\ (\sin x) = \sin^3 x$.

7. (d) $fog = gof$ for the given function.

8. (a) R = {(1, 2), (1, 3), (1, 1), (2, 2), (3, 3), (2, 1), (3, 1)}. Only 1 relation is possible.

9. (c) $(2 * 3) = 4 + 9 + 6 = 19$

$(19 * 3) = 361 + 9 + 57 = 427$

12. (c) B = {1, 2, 3, 4, 5, 6}

R = {(x, y) : y is divisible by x}

$(x, x) \in$ R [any number divisible by itself]

$\therefore$ It is reflexive.

Now, $(2, 4) \in$ R [as 4 is divisible by 2]

But $(4, 2) \notin$ R So, R is not symmetric

Let $(x, y), (y, z) \in$ R.

$\therefore$ z is divisible by x. Therefore, R is transitive.

13. (b) The total Number of reflexive relations is $2^{n^2 - n}$.

14. (d) f : R $\to$ R

$\therefore f(x_1) = x_1^2 - 2; f(x_2) = x_2^2 - 2$

$f(x_1) = f(x_2) \Rightarrow x_1^2 - 2 = x_2^2 - 2 \Rightarrow x_1^2 - x_2^2 = 0$

$(x_1 + x_2)(x_1 - x_2) = 0 \Rightarrow x_1 = \pm x_2$

$\therefore f(x_1) = f(x_2) \to$ not injective

$f(x)$ is not surjective. So, It is not bijective.

15. (b) f : A $\to$ B is onto if the range of 'f' is equal to the codomain of 'f'

16. (d) Let the side be x. Then, $P = 4x \Rightarrow x = \frac{P}{4}$

$\Rightarrow A = x^2 \Rightarrow A = \frac{P^2}{16}$

17. (c) In into function, if there exists atleast one element in B which do not have image at A.

18. (c) As $(x, x) \in$ R for all $x \in$ G. Thus, R is reflexive

Let $(x, y) \in$ R. x and y are same

$\therefore$ y and x are also of same $\Rightarrow (y, x) \in$ R. So, R is symmetric

Let $(x, y) \in$ R , $(y, z) \in$ R $\Rightarrow (x, z) \in$R Thus, R is transitive.

20. (b) R can not be reflexive as 'a' can not be brother of 'b'.

R cannot be symmetric as if $(a, b) \in$ R, then $(b, a) \in$ R, this is not possible.

R is transitive as, if a is brother of b and b is brother of c, then certainly a is brother of c.

21. (c) $f(x) + g(x) = \sqrt{x} + \sqrt{1-x}$ — domain [0, 1]

$f(x) - g(x) = \sqrt{x} - \sqrt{1-x}$ — domain [0, 1]

22. (a) $f(x) = \sin^{-1}\left[\log_2\left(\frac{x}{2}\right)\right]$ $-1 \le \log_2\left(\frac{x}{2}\right) \le 1 \Rightarrow 2^{-1} \le \frac{x}{2} \le 2 \Rightarrow 1 \le x \le 4 \Rightarrow x \in [1, 4]$

23. (c) When t is odd, values are all non-negative integers and if 't' is even values are set of all negative integers.

24. (d) $f(x + y) = f(x) + f(y) \Rightarrow f(x) = ax \Rightarrow f(1) = 5 \Rightarrow a(1) = 5 \therefore a = 5$

$$f(x) = 5x \Rightarrow \sum_{r=1}^{n} f(r) = 5\,(1 + 2 + 3 + \ldots\ldots n) = \frac{5n(n+1)}{2}$$

25. (b) $f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right) \forall\ x \in (-1, 1)$

Range $f(x) = \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ and Co-domain = X = $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

26. (b) $(x, x) \in R \ \forall \ x \in W$

$\therefore$ R is reflexive

Let $(x, y) \in$ R, then $(y, x) \in$ R as one letter is common

$\therefore$ R is symmetric.

28. (d) The number of mapping is $n \times n \times n \times$ ..... m times $= n^m$.

29. (b) In (I) and (III) horizontal lines meet the curve only at one point.

31. (a) $[\pi^2] = 9$ and $[-\pi^2] = 10$,

$f(x) = \cos 9x + \cos(-10)\, x = \cos 9x + \cos 10x$

$f\left(\frac{\pi}{2}\right) = -1$

32. (c) $f(x)$ is one-one and onto

$f(x) = [1 - (x-3)^4]^{1/7} \Rightarrow (x-3)^4 = 1 - (f(x))^7 \Rightarrow x = 3 + [1 - (f(x))^7]^{1/4}$

$f^{-1}(x) = 3 + (1 - x^7)^{1/4}$

33. (c) Since the graph lies between $f(-1) = -3$ and $f(2) = 2$. Hence the range is $[-3, 2]$

34. (a) $x^2 + 2mx - 1$ forms a parabola.

$mx - 1$ forms a line $\Rightarrow m \in (-\infty, 0)$

$m > 0$ not possible, as it is one-one function.

35. (d) $g(x)$ will be inverse of $f(x)$.

$y = (x+1)^2 \Rightarrow \sqrt{y} = x + 1 \Rightarrow x = \sqrt{y} - 1$

$f^{-1}(x) = -1 + \sqrt{x} : x \geq 0$

37. (a) $f(y) = f\left(\frac{x+2}{x-1}\right) = \frac{\frac{x+2}{x-1} + 2}{\frac{x+2}{n-1} - 1} = x$

38. (b) $f(x) = \frac{9^x}{9^3 + 3}, \quad f(1-x) = \frac{9^{1-x}}{9^{1-x} + 3}$

$$f(1-x) + f(x) = \frac{9^{1-x}}{9^{1-x}+3} + \frac{9^x}{9^3+3} = \frac{9}{9+3.9^x} + \frac{9^x}{9^x+3} = \frac{3}{3+9^x} + \frac{9^x}{9^x+3} = 1$$

$$\therefore f\left(\frac{1}{97}\right) + \ldots\ldots f\left(\frac{96}{97}\right) = 48.$$

39. (b) Since $A \times B \rightarrow B \times A$

$\therefore f$ is one-one

f is onto therefore, $f(x)$ is bijective function

40. (c) $aob = a$

$boa = b$ therefore, aob and boa are not commutative.

41. (c) 2 is the inverse of 1.

43. (d) As given that given triangles are similar

$\frac{3}{6} = \frac{4}{8}, \frac{5}{10} = \frac{1}{2}$

$\therefore$ The corresponding sides of T_1 and T_3 are in the same ratio, T_3 is related to T_1.

44. (b) The given relation is equivalence

$\therefore$ The elements in A related to right triangle with sides 3, 4 and 5. These are polygons with 3-sides. Hence, the set of all elements in A related to triangle T is the set of all triangles.

45. (d) $f : Z \to Z$ is given by

$f(x) = x^3 \Rightarrow f(x) = f(y) \Rightarrow x^3 = y^3 \Rightarrow x = y$

$\therefore f$ is injective,

Now, $2 \in Z$, But these does not exist any element x in domain Z.

$f(x) = x^3 = 2$ Therefore, f is not surjective.

46. (d) $f(x) = \begin{cases} 1, \text{ if } x > 0 \\ 0, \text{ if } x = 0 \\ -1, \text{ if } x < 0 \end{cases}$

$\therefore f(1) = f(2) = 1$ but $1 \neq 2$

$\therefore f$ is not one-one and f is not onto

As there does not exist any x in domain R such that $f(x) = -2$.

47. (d) $f : R \to R \to f(x) = x^{10}$

Let $x, y \in R$ such that $f(x) = f(y)$

$x^{10} = y^{10}$

$x = \pm y \Rightarrow f(x_1) = f(x_2)$

does not multiply that $x_1 = x_2$

$\therefore f$ is not one-one and $f(x)$ is not onto

48. (d) L.C.M. of a and $1 = a$

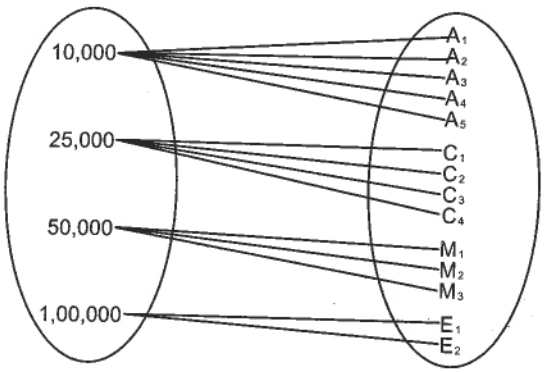
$a * 1 = a = 1 * a \; a \in N$

49. (c) R is reflexive on A.

$\therefore (a, a) \in R \; \forall \; a \in A$

The no. of ordered pair in R is n.

50. (a)



51. (i) (c) When fog is an identity function, then, $g = f^{-1}$

(ii) (a) $(fog)(x) = f(g(x)) = 2(g(x)) + 3 = 2(-x^2 + 1) + 3 = -2x^2 + 5$

(iii) (a) $(fog)(x) = g(f(x)) = \text{In}(1 - f(x)^2)$

$$\text{In}\left(1-\left(\sqrt{x+2}\right)^2\right) = \text{In}(1 - (x + 2)) = \text{In}\,(-1 - x)$$

$1 - f(x)^2 > 0$; $-x - 1 > 0$

$x < -1$ or in the interval $(-\infty, -1)$.

(iv) (c) $gof = \{(-2, 1), (0, 3)\}$; Range $= \{1, 3\}$

(v) (a) $F(x) = \text{In}\,(\text{In}(x)) \Rightarrow F(x) = \text{In}\,(u(x)$

Chain rule, $F'(x) = \left[\frac{d}{du}(\text{In}\,u)\right]\frac{du}{dx} = \frac{1}{u} \times \frac{1}{x} = \frac{1}{x} \cdot \frac{1}{\text{In}\,x}$