Chapter - 11 THREE-DIMENSIONAL GEOMETRY

STUDY NOTES

- Distance between two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$

  $|AB| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

- **Direction cosines of a line :** If α, β, γ are the direction angles of a line, then $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are called the direction cosines of the line. Usually direction cosines are denoted by l, m, n, i.e., $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$.

  Also, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, i.e., $l^2 + m^2 + n^2 = 1$.

  $$l = \cos\alpha = \frac{a}{\sqrt{a^2+b^2+c^2}},\ m = \cos\beta = \frac{b}{\sqrt{a^2+b^2+c^2}},\ n = \cos\gamma = \frac{c}{\sqrt{a^2+b^2+c^2}}.$$

- Shortest distance between two lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$ be two non-intesecting, non parallel lines is $\left|\frac{(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)}{(\vec{b}_1\times\vec{b}_2)}\right|$

- Let a plane passes through a point A with position vector $\vec{a}$, and be perpendicular to a direction along the vector $\vec{n}$, then vector equation of plane is $(\vec{r} - \vec{a})\cdot\vec{n} = 0$.

- Equation of plane passing through three non collinear points $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ is

  $$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

- Cartesian form of equation of plane passing through a point (x_1, y_1, z_1) and parallel to lines with direction ratios $<a_1, b_1, c_1>$ and $<a_2, b_2, c_2>$ is

  $$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

- Distance from a point $P(x_1, y_1, z_1)$ to the plane $ax + by + cz + d = 0$ is given by $\left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$

- Equation of a plane passing through a point whose position vector is $\vec{a}$ and perpendicular to the plane $\vec{r}\cdot\vec{n}_1 = d_1$ and $\vec{r}\cdot\vec{n}_2 = d_2$ is given by $(\vec{r}-\vec{a})\cdot(\vec{n}_1 \times \vec{n}_2) = 0$.

- Equation of a plane passing through two points P and Q, and perpendicular to the plane $\vec{r}\cdot\vec{n}_1 = d_1$ is given by $(\vec{r}-\vec{a})\cdot(\vec{n}_1 \times (\vec{b}-\vec{a})\ \vec{n}_2) = 0$, where $\vec{a}$ and $\vec{b}$ are position vectors of points P and Q respectively.

QUESTION BANK

MULTIPLE CHOICE QUESTIONS

1. Distance between two planes : $2x + 3y + 4z = 4$ and $4x + 6y + 8z = 12$ is :

(a) 2 units (b) 4 units (c) 8 units (d) $\frac{2}{\sqrt{29}}$ units

2. Equation of plane parallel to plane $x - 2y + 2z - 5 = 0$ and at a unit distance from the origin is:

(a) $x - 2y + 2z - 3 = 0$ (b) $x - 2y + 2z + 1 = 0$ (c) $x - 2y + 2z - 1 = 0$ (d) $x - 2y + 2z + 5 = 0$

3. Unit normal vector to plane $2x - 2y - z - 5 = 0$ is :

(a) $2\hat{i} - 2\hat{j} - \hat{k}$ (b) $\pm\frac{1}{3}(2\hat{i} - 2\hat{j} - \hat{k})$ (c) $\frac{1}{3}(2\hat{i} - 2\hat{j} - \hat{k})$ (d) $-\frac{1}{3}(2\hat{i} - 2\hat{j} - \hat{k})$

4. The distance between two parallel planes : $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is :

(a) $\frac{5}{2}$ units (b) $\frac{7}{2}$ units (c) $\frac{9}{2}$ units (d) $\frac{3}{2}$ units

5. The distance of plane $\vec{r}\cdot\left(\frac{-2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right) = 1$ from the origin is :

(a) 1 unit (b) 7 units (c) $\frac{1}{7}$ units (d) 3 units

6. The distance of plane $2x - 3y + 6z + 7 = 0$ from point $(2, -3, -1)$ is :

(a) 4 units (b) 3 units (c) 1 unit (d) $\frac{1}{5}$ units

7. The planes : $2x - y + 4z = 5$ and $5x - 2.5y + 10z = 6$ are

(a) perpendicular (b) parallel

(c) intersect y-axis (d) passes through $\left(0, 0, \frac{5}{4}\right)$

8. If α, β, γ be the angles which a line makes with the positive direction of co-ordinate axes, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma =$

(a) 2 (b) 1 (c) 3 (d) 0

9. The angle between the lines $2x = 3y = -z$ and $6x - y = -4z$ is :

(a) 90° (b) 0° (c) 30° (d) 45°

10. P is a point on the line segment joining the points $(3, 2, -1)$ and $(6, 2, -2)$. If x co-ordinate of P is 5, then its y co-ordinate is :

(a) 2 (b) 1 (c) –1 (d) –2

11. The distance of a point P(a, b, c) from x-axis is :

(a) $\sqrt{a^2 + c^2}$ (b) $\sqrt{a^2 + b^2}$ (c) $\sqrt{b^2 + c^2}$ (d) $b^2 + c^2$

12. The point of intersection of lines $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is :

(a) $(-1, -1, -1)$ (b) $(-1, -1, 1)$ (c) $(1, -1, -1)$ (d) $(-1, 1, -1)$

13. The angle between two lines $\frac{x}{2} = \frac{y}{2} = \frac{z}{-1}$ and $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{2}$ is :

(a) $\cos^{-1}\left(\frac{4}{9}\right)$ (b) $\cos^{-1}\left(\frac{1}{3}\right)$ (c) $\cos^{-1}\left(\frac{2}{9}\right)$ (d) $\cos^{-1}\left(\frac{5}{9}\right)$

14. The shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ is :

(a) $\sqrt{30}$ (b) $2\sqrt{30}$ (c) $5\sqrt{30}$ (d) $3\sqrt{30}$

15. The image of the point (–1, 3, 4) in the plane $x - 2y = 0$ is :
(a) $\left(-\frac{17}{3}, \frac{-19}{3}, 4\right)$ (b) (15, 11, 4) (c) $\left(\frac{-17}{3}, \frac{-19}{3}, 1\right)$ (d) $\left(\frac{9}{5}, \frac{-13}{5}, 4\right)$

16. The angle between the lines whose direction cosines satisfy the relations, $l + m + n = 0$ and $3\,lm - 5mn + 2nl = 0$ is :
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\cos^{-1}\left(\frac{3}{\sqrt{16}}\right)$

17. The foot of the perpendicular drawn from the point (1, 0, 2) on the line $\frac{x+1}{3} = \frac{y-1}{-2} = \frac{z+1}{-1}$ is :
(a) $\left(\frac{1}{2}, 1, \frac{-3}{2}\right)$ (b) $\left(\frac{2}{3}, 1, -1\right)$ (c) $\left(\frac{2}{3}, \frac{-1}{2}, 2\right)$ (d) (1, –2, –1)

18. If O is the origin and OP = 3 with direction ratios –1, 2, –2, then co-ordinates of P are
(a) (1, 2, 2) (b) (–1, 2, –2) (c) (–3, 6, –9) (d) $\left(\frac{-1}{3}, \frac{2}{3}, \frac{-2}{3}\right)$

19. The equation of the plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$, parallel to x-axis is :
(a) $y - 3z + 6 = 0$ (b) $3y - z + 6 = 0$ (c) $y + 3z + 6 = 0$ (d) $3y - 2z + 6 = 0$

20. A plane passes through (1, –2, 1) and is perpendicular to two planes $2x - 2y + z = 0$ and $x - y + 2z = 4$. The distance of the plane from the point, (1, 2, 2) is :
(a) 0 (b) 1 (c) $\sqrt{2}$ (d) $2\sqrt{2}$

21. The equation of a plane which passes through (2, –3, 1) and is normal to the line joining the points (3, 4, –1) and (2, –1, 5) is given by
(a) $x + 5y - 6z + 19 = 0$ (b) $x - 5y + 6z - 19 = 0$
(c) $x + 5y + 6z + 19 = 0$ (d) $x - 5y - 6z - 19 = 0$

22. The angle between two planes $2x - y + z = 6$ and $x + 2y + 3z = 3$ is :
(a) $\cos^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{7}}\right)$ (b) $\cos^{-1}\left(\frac{1}{2}\sqrt{\frac{2}{7}}\right)$ (c) $\cos^{-1}\left(\frac{1}{2}\sqrt{\frac{3}{7}}\right)$ (d) $\cos^{-1}\left(\frac{1}{2}\sqrt{\frac{4}{7}}\right)$

23. The plane through two points A(–1, 1, 1) and B(1, –1, 1) and perpendicular to the plane $x + 2y + 2z = 5$ has equation :
(a) $2x + 2y + 3z = 0$ (b) $2x + 2y + 3z + 3 = 0$
(c) $2(x + y) = 3(z - 1)$ (d) $3x + 3y + 3z - 1 = 0$

24. The equation of the plane through (2, 3, 4) and parallel to the plane $x + 2y + 4z = 5$ is :
(a) $x + 2y + 4z = 10$ (b) $x + 2y + 4z = 3$ (c) $x + y + 2z = 2$ (d) $x + 2y + 4z = 24$

25. The planes $x = cy + bz$, $y = az + cx$, $z = bx + ay$ pass through one line, if
(a) $a + b + c = 0$ (b) $a + b + c = 1$
(c) $a^2 + b^2 + c^2 = 1$ (d) $a^2 + b^2 + c^2 + 2abc = 1$

26. The length of the perpendicular from the origin to the plane $3x + 4y + 12z = 52$ is :
(a) 3 (b) –4 (c) 5 (d) 4

27. The intercepts of the plane $5x - 3y + 6z = 60$ on the co-ordinate axes are :
(a) (10, 20, – 10) (b) (10, –20, 12) (c) (12, –20, 10) (d) (12, 20, –10)

28. If the planes $x + 2y + kz = 0$ and $2x + y - 2z = 0$ are at right angles, then the value of k is :
(a) $\frac{-1}{2}$ (b) $\frac{1}{2}$ (c) –2 (d) 2

29. Distance of the point (α, β, γ) from y-axis is :

(a) β (b) $|\beta|$ (c) $|\beta| + |\gamma|$ (d) $\sqrt{\alpha^2+\gamma^2}$

30. The reflection of the point (α, β, γ) in the xy-plane is :

(a) (α, β, 0) (b) (0, 0, γ) (c) (–α, –β, γ) (d) (α, β, – γ)

31. The plane $2x - 3y + 6z - 11 = 0$ makes an angle $\sin^{-1}\alpha$ with x-axis. The value of α is equal to :

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{\sqrt{2}}{3}$ (c) $\frac{2}{7}$ (d) $\frac{3}{7}$

32. The area of quadrilateral ABCD, where A(0, 4, 1) B(2, 3, –1), C(4, 5, 0) and D(2, 6, 2) is equal to :

(a) 9 sq. units (b) 18 sq. units (c) 27 sq. units (d) 81 sq. units

33. The ratio in which the join of (2, 1, 5) and (3, 4, 3) is divided by the plane $(x + y - z) = \frac{1}{2}$ is :

(a) 3 : 5 (b) 5 : 7 (c) 1 : 3 (d) 4 : 5

34. The angle between the two planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$ is :

(a) $\cos^{-1}\left(\frac{4}{21}\right)$ (b) $\cos^{-1}\left(\frac{2}{21}\right)$ (c) $\cos^{-1}\left(\frac{1}{21}\right)$ (d) $\cos^{-1}\left(\frac{5}{21}\right)$

35. The foot of the perpendicular from the point (7, 14, 15) to the plane $2x + 4y - z = 2$ are :

(a) (1, 2, 8) (b) (3, 2, 8) (c) (5, 10, 6) (d) (9, 18, 4)

36. The locus represented by $xy + yz = 0$ is :

(a) A pair of perpendicular lines (b) A pair of parallel lines

(c) A pair of parallel planes (d) A pair of perpendicular planes

37. The sine of the angle between the straight line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ and the plane $2x - 2y + z = 5$ is :

(a) $\frac{10}{6\sqrt{5}}$ (b) $\frac{4}{5\sqrt{2}}$ (c) $\frac{2\sqrt{3}}{5}$ (d) $\frac{\sqrt{2}}{10}$

38. If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then the value of k is :

(a) $\frac{3}{2}$ (b) $\frac{9}{2}$ (c) $\frac{-2}{9}$ (d) $\frac{-3}{2}$

39. Two lines $L_1 : x = 5, \frac{y}{3-\alpha} = \frac{z}{-2}$, $L_2 : x = \alpha, \frac{y}{-1} = \frac{z}{2-\alpha}$ are coplanar. Then, α can take value(s) :

(a) 1, 4, 5 (b) 1, 2, 5 (c) 3, 4, 5 (d) 2, 4, 5

40. The perpendicular distance of P(1, 2, 3) from the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$ is :

(a) 7 (b) 5 (c) 0 (d) 6

41. If the line through the point A(k, 1, –1) and B (2k, 0, 2) is perpendicular to the line through the points B and C(2 + 2k, k, 1), then what is the value of k?

(a) –1 (b) 1 (c) –3 (d) 3

42. A line makes the same angle θ with each of the X and Z-axes. If the angle β, which it makes with Y-axis, is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals :

(a) $\frac{2}{5}$ (b) $\frac{1}{5}$ (c) $\frac{3}{5}$ (d) $\frac{2}{3}$

43. The coordinates of the point where the line through the points A(3, 4, 1) and B(5, 1, 6) crosses the XY-plane are:

(a) $\left(\frac{13}{5}, \frac{23}{5}, 0\right)$ (b) $\left(-\frac{13}{5}, \frac{23}{5}, 0\right)$ (c) $\left(\frac{13}{5}, \frac{-23}{5}, 0\right)$ (d) $\left(-\frac{13}{5}, \frac{-23}{5}, 0\right)$

44. Gives the line L : $\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-3}{-1}$ and the plane π : $x-2y-z=0$, of the following assertions, the only one that is always true is :

(a) L is ⊥ to π (b) L lies in π (c) L is not parallel to π (d) None of these

45. The angle between the line $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ and the plane $10x+2y-11z=3$ is :

(a) $\sin^{-1}\left(\frac{8}{21}\right)$ (b) $\sin^{-1}\left(\frac{5}{21}\right)$ (c) $\sin^{-1}\left(\frac{7}{21}\right)$ (d) $\sin^{-1}\left(\frac{1}{21}\right)$

46. What is the value of n so that the angle between the lines having direction ratios (1, 1, 1) and (1, –1, n) is 60°?

(a) $\sqrt{3}$ (b) $\sqrt{6}$ (c) 3 (d) None of these

47. The equation of the right bisector plane of the segment joining (2, 3, 4) and (6, 7, 8) is :

(a) $x+y+z+15=0$ (b) $x+y+z-15=0$ (c) $x-y+z-15=0$ (d) None of these

48. The coordinates of the point where the line joining the points (2, –3, 1) and (3, –4, –5) cuts the plane $2x+y+z=7$ is :

(a) (1, 2, –7) (b) (1, –2, 7) (c) (–1, –2, 7) (d) (1, 2, 7)

49. The equation of line of intersection of planes $4x+4y-5z=12$, $8x+12y-13z=32$ can be written as :

(a) $\frac{x-1}{2}=\frac{y+2}{-3}=\frac{z}{4}$ (b) $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z}{4}$ (c) $\frac{x}{2}=\frac{y+1}{3}=\frac{z-2}{4}$ (d) $\frac{x}{2}=\frac{y}{3}=\frac{z-2}{4}$

50. If the line $\frac{x-4}{1}=\frac{y-2}{1}=\frac{z-k}{2}$ lies in the plane $2x-4y+z=7$, then the value of k is :

(a) 4 (b) –7 (c) 7 (d) No real value

51. What is the angle between the lines $\frac{x-2}{1}=\frac{y+1}{-2}=\frac{z+2}{1}$ and $\frac{x-1}{1}=\frac{2y+3}{3}=\frac{z+5}{2}$?

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) None of these

52. The point of intersection of the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}$ is :

(a) $\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$ (b) $\left(\frac{1}{2},-\frac{1}{2},-\frac{3}{2}\right)$ (c) $\left(\frac{1}{3},-\frac{1}{3},-\frac{2}{3}\right)$ (d) None of these

53. The line, $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}$ intersects the curve $xy=c^2$, $z=0$ if c is equal to :

(a) ±1 (b) $\pm\frac{1}{3}$ (c) $\pm\sqrt{5}$ (d) None of these

54. The direction cosines of the line joining the points A(6, –7, –1) and B(2, –3, 1) is :

(a) $\left[\pm\frac{2}{3},\pm\frac{2}{3},\pm\frac{1}{3}\right]$ (b) $\left[\pm\frac{2}{3},\pm\frac{1}{3},\pm\frac{1}{3}\right]$ (c) $\left[\pm\frac{1}{3},\pm\frac{2}{3},\pm\frac{2}{3}\right]$ (d) $\left[\pm\frac{1}{3},\pm\frac{1}{3},\pm\frac{2}{3}\right]$

55. A line segment has length 63 and direction rations are 3, –2, 6. If the line makes an obtuse angle with x-axis, the components of the line vector are :

(a) 27, –18, 54 (b) –27, 18, 54 (c) –27, 18, –54 (d) 27, –18, –54

56. The coordinates of the point where the line segment joining A(5, 1, 6) and B(3, 4, 1) crosses the yz plane are :

(a) $\left(0,-\frac{17}{2},-\frac{17}{2}\right)$ (b) $\left(0,\frac{17}{2},-\frac{13}{2}\right)$ (c) $\left(0,\frac{13}{2},\frac{-17}{2}\right)$ (d) $\left(0,\frac{-13}{2},\frac{-17}{2}\right)$

57. If a line makes angle α, β and γ with the coordinate axes respectively then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma =$

(a) 1 (b) –1 (c) 2 (d) –2

58. The projection of the line segment joining the points A(–1, 0, 3) and B(2, 5, 1) on the line whose direction ratios are proportional to 6, 2, 3 is :

(a) $\frac{10}{7}$ (b) $\frac{22}{7}$ (c) $\frac{18}{7}$ (d) $\frac{15}{7}$

59. The angle between the lines whose direction ratios are proportional to 1, 1, 2 and $\sqrt{3}-1, -\sqrt{3}-1$, 4 is :

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

60. The projection of a line segment on the coordinate axes are 2, 3 and 6. Then, the length of the line segment is:

(a) 5 (b) 6 (c) 7 (d) 1

61. A line makes the same angle θ, with each of the x and z-axes. If the angle β, which it makes with y-axis is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals :

(a) $\frac{2}{5}$ (b) $\frac{1}{5}$ (c) $\frac{3}{5}$ (d) $\frac{2}{3}$

62. A line AB in three dimensional space marks angles 45° and 120° with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals :

(a) 60° (b) 75° (c) 45° (d) 30°

63. If the planes $\vec{r}\cdot(2\hat{i} - \hat{j} + 2\hat{k}) = 4$ and $\vec{r}\cdot(3\hat{i} + 2\hat{j} + \lambda\hat{k}) = 3$ are perpendicular, then $\lambda =$

(a) 2 (b) –2 (c) 3 (d) –3

64. If the lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d'$ are perpendicular, then

(a) $aa' + cc' = -1$ (b) $aa' + cc' = 1$ (c) $aa' + bb' = cc'$ (d) $ab + cd = a'b' + c'd'$

65. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$, is :

(a) 0° (b) 30° (c) 45° (d) 90°

66. The line of intersection of the planes $\vec{r}\cdot(3\hat{i} - \hat{j} + \hat{k}) = 1$ and $\vec{r}\cdot(\hat{i} + 4\hat{j} - 2\hat{k}) = 2$ is parallel to the vector.

(a) $2\hat{i} + 7\hat{j} + 13\hat{k}$ (b) $-2\hat{i} - 7\hat{j} + 13\hat{k}$ (c) $2\hat{i} + 7\hat{j} + 13\hat{k}$ (d) $-2\hat{i} + 7\hat{j} + 13\hat{k}$

INPUT TEXT BASED MCQ's

67. Consider the lines

Line (l_1) : $\frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}$ and line (l_2) : $\frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$.

Answer the following questions :

(i) The shortest distance between line (l_1) and line (l_2) is :

(a) 0 (b) $\frac{17}{\sqrt{3}}$ (c) $\frac{17}{5\sqrt{3}}$ (d) $\frac{41}{5\sqrt{3}}$

(ii) Unit vector perpendicular to both line (l_1) and line (l_2) is :

(a) $\frac{7\hat{i}-7\hat{j}-\hat{k}}{\sqrt{99}}$ (b) $\frac{-\hat{i}+7\hat{j}+5\hat{k}}{5\sqrt{3}}$ (c) $\frac{-\hat{i}+7\hat{j}+7\hat{k}}{\sqrt{99}}$ (d) $\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$

(iii) The distance of the point (1, 1, 1) from the plane passing through (–1, –2, –1) and whose normal is perpendicular to both lines l_1 and l_2 is :

(a) $\frac{2}{\sqrt{75}}$ (b) $\frac{7}{\sqrt{75}}$ (c) $\frac{13}{\sqrt{75}}$ (d) $\frac{23}{\sqrt{75}}$

(iv) If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then the value of k is :

(a) $\frac{3}{2}$ (b) $\frac{9}{2}$ (c) $\frac{-2}{9}$ (d) $\frac{-3}{2}$

(v) The equation of the plane containing the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$

Which is perpendicular to the plane containing the lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is :

(a) $x+2y-2z=0$ (b) $3x+2y-2z=0$ (c) $x-2y+z=0$ (d) $5x+2y-4z=0$

68. Two cars are running along the lines P and Q at a speed more than the permissible speed on the road, i.e., $\vec{r}=\lambda(\hat{i}-\hat{j}+2\hat{k})$ and $\vec{r}=3\hat{i}+3\hat{j}+\mu(2\hat{i}+\hat{j}+\hat{k})$ respectively.

Answer the following questions :

(i) The cartesian equation of the line along which car P is running is :

(a) $\frac{x}{2}=\frac{y}{-1}=\frac{z}{1}$ (b) $\frac{x}{1}=\frac{y}{-1}=\frac{z}{2}$ (c) $\frac{x}{-1}=\frac{y}{2}=\frac{z}{1}$ (d) $\frac{x}{-1}=\frac{y}{1}=\frac{z}{2}$

(ii) The direction cosines of line along which car P is running are :

(a) $<\frac{1}{\sqrt{6}},\frac{2}{\sqrt{6}},\frac{1}{\sqrt{6}}>$ (b) $<\frac{1}{\sqrt{6}},\frac{-1}{\sqrt{6}},\frac{1}{\sqrt{6}}>$ (c) $<\frac{1}{\sqrt{6}},\frac{-1}{\sqrt{6}},\frac{2}{\sqrt{6}}>$ (d) $<-1,\frac{-1}{\sqrt{6}},\frac{2}{\sqrt{6}}>$

(iii) The direction ratios of the line along which car Q is running, are

(a) <1, 1, 2> (b) <2, 1, 1> (c) <2, –1, 1> (d) <1, 2, –1>

(iv) The shortest distance between the given lines is :

(a) 0 units (b) $\sqrt{3}$ units (c) $2\sqrt{2}$ units (d) 4 units

(v) The cars will meet with an accident at the point

(a) (1, 1, –2) (b) (1, –1, 2) (c) (–1, –1, 2) (d) (2, 1, 1)

ANSWERS

1. (d)	**2.** (a)	**3.** (b)	**4.** (b)	**5.** (a)	**6.** (c)	**7.** (b)	**8.** (a)	**9.** (a)	**10.** (a)
11. (c)	**12.** (a)	**13.** (a)	**14.** (d)	**15.** (d)	**16.** (b)	**17.** (a)	**18.** (b)	**19.** (a)	**20.** (d)
21. (a)	**22.** (c)	**23.** (c)	**24.** (d)	**25.** (a)	**26.** (d)	**27.** (c)	**28.** (d)	**29.** (d)	**30.** (d)
31. (c)	**32.** (a)	**33.** (b)	**34.** (a)	**35.** (a)	**36.** (d)	**37.** (d)	**38.** (b)	**39.** (a)	**40.** (a)
41. (d)	**42.** (c)	**43.** (a)	**44.** (b)	**45.** (a)	**46.** (b)	**47.** (b)	**48.** (b)	**49.** (b)	**50.** (c)
51. (a)	**52.** (b)	**53.** (c)	**54.** (a)	**55.** (c)	**56.** (b)	**57.** (b)	**58.** (b)	**59.** (a)	**60.** (c)
61. (c)	**62.** (a)	**63.** (b)	**64.** (a)	**65.** (d)	**66.** (d)				

67. (i) (c) (ii) (d) (iii) (c) (iv) (b) (v) (c)

68. (i) (b) (ii) (c) (iii) (b) (iv) (a) (v) (b)

Hints to Some Selected Questions

1. (d) Distance between two parallel planes

$Ax+By+Cz=d_1$ and $Ax+By+Cz=d_2$ is

$$\left|\frac{d_1-d_2}{\sqrt{A^2+B^2+C^2}}\right|$$

$$\therefore \text{Distance}=\left|\frac{4-6}{\sqrt{2^2+3^2+4^2}}\right|=\frac{2}{\sqrt{29}} \text{ units}$$

2. (a) Plane parallel to $x - 2y + 2z - 5 = 0$ is $x - 2y + 2z + k = 0$

Distance from origin = 1

$\Rightarrow \frac{k}{\sqrt{9}} = 1 \Rightarrow k = \pm 3$

$\therefore x - 2y + 2z - 3 = 0.$

3. (b) Direction ratios of normal vector are (2, −2, −1)

Unit normal vector is $\frac{2\hat{i} - 2\hat{j} - \hat{k}}{\pm 3} = \pm\frac{1}{3}(2\hat{i} - 2\hat{j} - \hat{k})$

4. (b) Distance = $\frac{\left|8 + \frac{5}{2}\right|}{\sqrt{4+1+4}} = \frac{7}{2}$ units

5. (a) Required distance = $\frac{\left|(0\hat{i} + 0\hat{j} + 0\hat{k})\cdot\left(-\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right) - 1\right|}{\sqrt{\left(\frac{4}{49} + \frac{9}{49} + \frac{36}{49}\right)}} = 1$ unit

6. (c) Distance = $\frac{(2\times 2 + (-3)(-3) + 6\times -1)}{\sqrt{4+9+36}} = \frac{7}{7} = 1$

7. (b) $\cos\theta = \left|\frac{(2\times 10) + (-1\times -5) + (4\times 20)}{\sqrt{2^2 + (-1)^2 + (4^2)}\sqrt{10^2 + (-5)^2 + 20^2}}\right|$

$\cos\theta = \left|\frac{20+5+80}{\sqrt{21}\sqrt{525}}\right| = \left|\frac{105}{\sqrt{21}\times\sqrt{21}\times 5}\right| = 1$

So, $\cos\theta = 1 \Rightarrow \theta = 0$. Therefore, the planes are parallel.

8. (a) Since, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 3 - 1 = 2.$

10. (a) Let P divides the line segment in the ratio of $\lambda : 1$,

x-coordinate of the point P may be expressed as $x = \frac{6\lambda+3}{\lambda+1}$ giving $\frac{6\lambda+3}{\lambda+1} = 5$

so that $\lambda = 2$. Thus, y-coordinate of P is $\frac{2\lambda+2}{\lambda+1} = 2$

11. (c) The required distance is the distance of P(a, b, c) from Q(a, 0, 0), which is $\sqrt{b^2+c^2}$.

12. (a) Both the lines are satisfied by (−1, −1, −1).

14. (d) S.D. = $\frac{\begin{vmatrix} 6 & 15 & -3 \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix}}{\sqrt{(-4-2)^2 + (12+3)^2 + (6-3)^2}} = \frac{270}{\sqrt{270}} = \sqrt{270} = 3\sqrt{30}.$

17. (a) Any point on the given line is of the form

Q = (−1 + 3λ, 2 − 2λ, −1 −λ). Let P = (1, 0, 2). $\overrightarrow{PQ}$ is perpendicular to the given line

So, (3λ − 2, 2 − 2λ, −3 − λ), (3, −2, − 1) = 0

$\Rightarrow 9\lambda - 6 - 2(2 - 2\lambda) - 1(-3 - \lambda) = 0 \Rightarrow \lambda = \frac{1}{2}$

Therefore, Q = $\left(\frac{1}{2}, 1, -\frac{3}{2}\right)$ is the foot of the perpendicular from P onto the line.

18. (b) Co-ordinates of P are (lr, mr, nr)

Here, $l = \dfrac{-1}{\sqrt{1^2+2^2+2^2}} = \dfrac{-1}{3}$, $m = \dfrac{2}{3}$, $n = \dfrac{-2}{3}$

and $r = 3$

$\therefore$ co-ordinates of P are (–1, 2, –2).

19. (a) The equation of the plane through the intersection of plane $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ is

$(x + y + z - 1) + \lambda\,(2x + 3y - z + 4) = 0$

or $(1 + 2\lambda)\,x + (1 + 3\lambda)\,y + (1 - \lambda)z + 4\lambda - 1 = 0$

Since the plane parallel to x-axis,

$\therefore\ 1 + 2\lambda = 0 \Rightarrow \lambda = \dfrac{-1}{2}$.

Hence, the required equation will be $y - 3z + 6 = 0$.

20. (d) The plane is $a(x - 1) + b(y + 2) + c(z - 1) = 0$

where $2a - 2b + c = 0$ and $a - b + 2c = 0$

$\Rightarrow \dfrac{a}{1} = \dfrac{b}{1} = \dfrac{c}{0} = k$

So, the equation of plane $x + y + 1 = 0$

$\therefore$ Distance from the point (1, 2, 2) $= \dfrac{1+2+1}{\sqrt{1^2+1^2}} = 2\sqrt{2}$.

21. (a) Obviously $(x - 2) + 5(y + 3) - 6\,(z - 1) = 0$

$\Rightarrow x + 5y - 6z + 19 = 0$

22. (c) We have equation of planes are $2x - y + z = 6$ and $x + 2y + 3z = 3$

$\therefore\ \cos\theta = \dfrac{2+(-2)+3}{\sqrt{2^2+(-1)^2+1^2}\sqrt{1^2+2^2+3^2}} = \dfrac{3}{\sqrt{6}\cdot\sqrt{14}} = \sqrt{\dfrac{3}{28}}$

$\Rightarrow \theta = \cos^{-1}\left(\dfrac{1}{2}\sqrt{\dfrac{3}{7}}\right)$.

23. (c) Any plane through (–1, 1, 1) is $A(x + 1) + B(y - 1) + C(z - 1) = 0$

$\therefore$ It passes through (1, –1, 1)

$\Rightarrow 2A - 2B + 0C = 0$ $\quad\because$ It is $\perp$ to $x + 2y + 2z = 5$

$\Rightarrow A + 2B + 2C = 0$

Solving $\dfrac{A}{-4} = \dfrac{B}{-4} = \dfrac{C}{6}$

$\therefore$ Required plane is $-4(x + 1) - 4\,(y - 1) + 6(z - 1) = 0$

$\Rightarrow 2(x + 1) + 2(y - 1) - 3(z - 1) = 0 \Rightarrow 2(x + y) = 3\,(z - 1)$.

24. (d) The plane will be $x + 2y + 4z = 2 \times 1 + 3 \times 2 + 4 \times 4$ or $x + 2y + 4z = 24$.

25. (d) The planes are concurrent, therefore,

$$\begin{vmatrix} -1 & c & b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0 \Rightarrow a^2 + b^2 + c^2 + 2abc = 1.$$

26. (d) Length $= \left|\dfrac{-52}{\sqrt{9+16+144}}\right| = \left|\dfrac{-52}{13}\right| = 4$.

27. (c) $\frac{5x}{60} - \frac{3y}{60} + \frac{6z}{60} = 1 \Rightarrow \frac{x}{12} - \frac{y}{20} + \frac{z}{10} = 1.$

Hence, the intercepts are (12, –20, 10).

28. (d) We have, the planes are $x + 2y + kz = 0$ and $2x + y - 2z = 0$

Then, $2 \times 1 + 1 \times 2 + k \times -2 = 0 \Rightarrow k = 2.$

29. (d) The given point is (α, β, γ)

Any point on y-axis = $(0, \beta, 0)$

$\therefore$ Required distance = $\sqrt{(\alpha-0)^2 + (\beta-\beta)^2 + (\gamma-0)^2} = \sqrt{\alpha^2+\gamma^2}$

30. (d) Reflection of point (α, β, γ) in xy-plane is $(\alpha, \beta, -\gamma)$.

31. (c) D.R's of the normal to the plane $2x - 3y + 6z - 11 = 0$ are 2, –3, 6.

Direction ratios of x-axis are 1, 0, 0

$\therefore$ Angle between plane and line is

$$\sin\theta = \frac{2\times1 - 3\times0 + 6\times0}{\sqrt{2^2+(-3)^2+6^2}\cdot\sqrt{1^2+0^2+0^2}} = \frac{2}{7}.$$

34. (a) $\cos\theta = \left|\frac{(2\hat{i}+\hat{j}-2\hat{k})\cdot(3\hat{i}-6\hat{j}-2\hat{k})}{\sqrt{4+1+4}\sqrt{9+36+4}}\right| = \frac{4}{21}$

Hence, $\theta = \cos^{-1}\left(\frac{4}{21}\right).$

35. (a) Length of the $\perp$ from the point (x_1, y_1, z_1) to the plane $ax + by + cz + d = 0$ is $\frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$

and the co-ordinate (α, β, γ) of the foot of the $\perp$ are given by $\frac{\alpha-x_1}{a} = \frac{\beta-y_1}{b} = \frac{\gamma-z_1}{c}$

$$= -\left(\frac{ax_1+by_1+cz_1+d}{a^2+b^2+c^2}\right) \quad ...(i)$$

$x_1 = 7, y_1 = 14, z_1 = 5, a = 2, b = 4, c = -1$ and $d = -2$

By putting these value in (i), we get

$\frac{\alpha-7}{2} = \frac{\beta-14}{4} = \frac{\gamma-5}{-1} = \frac{-63}{21} \Rightarrow \alpha = 1, \beta = 2$ and $\gamma = 8.$

Hence, foot of $\perp$ is (1, 2, 8).

36. (d) Given $xy + yz = 0 \Rightarrow y\cdot(x+z) = 0 \Rightarrow y = 0$ or $x + z = 0$

Here, $y = 0$ is one plane and $x + z = 0$ is another plane.

So, it is a pair of perpendicular planes.

37. (d) Given line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ and plane : $2x - 2y + z = 5$. D, ratios of the line are 3, 4, 5.

and D, ratios of the normal to the plane are (2, –2, 1)

$$\therefore \sin\theta = \frac{3\times2+4\times-2+5\times1}{\sqrt{9+16+25}\sqrt{4+4+1}} = \frac{6-8+5}{\sqrt{50\times3}} = \frac{\sqrt{2}}{10}.$$

38. (b) Let A = (1, –1, 1), B = (3, k, 0), $\vec{n}_1 = (2, 3, 4)$ and $\vec{n}_2 = (1, 2, 1)$. The lines intersect

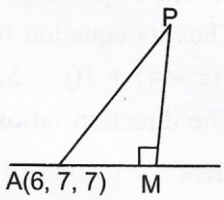
$\Rightarrow \overrightarrow{AB}, \vec{n}_1, \vec{n}_2$ are coplanar.

Hence, $\begin{vmatrix} 2 & k+1 & -1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow 2(3-8)-(k+1)(2-4)-1(4-3)=0$

$\Rightarrow -10+2k+2-1=0 \Rightarrow k=\frac{9}{2}$.

40. (a) The point A(6, 7, 7) is on the line.



Let the perpendicular from P meet the line in M.

Then, $AP^2=(6-1)^2+(7-2)^2+(7-3)^2=66$

Also, AM = Projection of AP on line

$\left(\text{D.C.'s } \frac{3}{\sqrt{17}}, \frac{2}{\sqrt{17}}, \frac{-2}{\sqrt{17}}\right)$

$\Rightarrow (6-1), \frac{3}{\sqrt{17}}+(7-2)\cdot\frac{2}{\sqrt{17}}+(7-3)\cdot\frac{-2}{\sqrt{17}}=\sqrt{17}$

Perpendicular distance $=\sqrt{AP^2-AM^2}=\sqrt{66-17}=\sqrt{49}=7$.

42. (c) Let l, m and n be the direction cosines.

Then, $l=\cos\theta$, $m=\cos\beta$, $n=\cos\theta$

we have $l^2+m^2+n^2=1$

$\Rightarrow \cos^2\theta+\cos^2\beta+\cos^2\theta=1 \Rightarrow 2\cos^2\theta+1-\sin^2\beta=1$

$\Rightarrow 2\cos^2\theta-\sin^2\beta=0 \Rightarrow 2\cos^2\theta-3\sin^2\beta=0 \Rightarrow \tan^2\theta=\frac{2}{3}$

$\therefore \cos^2\theta=\frac{1}{1+\tan^2\theta}=\frac{1}{1+2/3}=\frac{3}{5}$.

44. (b) Since $3(1)+2(-2)+(-1)(-1)=3-4+1=0$

$\therefore$ Given line is $\perp$ to the normal to the plane, i.e., given line is parallel to the given plane.

Also, (1, –1, 3) lies on the plane $x-2y-z=0$ if $1-2(-1)-3=0$, i.e., $1+2-3=0$

which is true $\therefore$ L lies in plane π.

45. (a) Let θ be the angle between the line and the normal to the plane. Converting the given equations into vector form, we have

$\vec{r}=(-\hat{i}+3\hat{k})+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$ and

$\vec{r}\cdot(10\hat{i}+2\hat{j}-11\hat{k})=3$

Here, $\vec{b}=2\hat{i}+3\hat{j}+6\hat{k}$ and $\vec{n}=10\hat{i}+2\hat{j}-11\hat{k}$

$$\sin\phi=\left|\frac{(2\hat{i}+3\hat{j}+6\hat{k})\cdot(10\hat{i}+2\hat{j}-11\hat{k})}{\sqrt{2^2+3^2+6^2}\sqrt{10^2+2^2+11^2}}\right|=\left|\frac{-40}{7\times15}\right|=\left|\frac{-8}{21}\right|=\frac{8}{21} \text{ or } \phi=\sin^{-1}\left(\frac{8}{21}\right).$$

46. (b) If (l_1, m_1, n_1) and (l_2, m_2, n_2) are the direction ratios then angle between the lines is

$$\cos\theta=\frac{l_1l_2+m_1m_2+n_1n_2}{\sqrt{l_1^2+m_1^2+n_1^2}\sqrt{l_2^2+m_2^2+n_2^2}}$$

Here, $l_1=1$, $m_1=1$, $n_1=1$ and

$l_2=1$, $m_2=-1$, $n_2=n$ and $\theta=60°$.

$$\therefore \cos 60° = \frac{1\times1+1\times(-1)+1\times n}{\sqrt{1^2+1^2+1^2}\times\sqrt{1^2+1^2+n^2}}$$

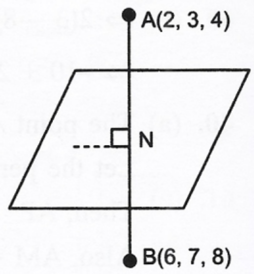
$$\Rightarrow \frac{1}{2}=\frac{n}{\sqrt{3}\sqrt{2+n^2}} \Rightarrow 3(2+n^2)=4n^2 \Rightarrow n^2=6 \Rightarrow n=\pm\sqrt{6}.$$

47. (b) If the given points be A(2, 3, 4) and B (6, 7, 8), then their mid-point N(4, 5, 6) must lie on the plane. The direction ratios of AB are 4, 4, 4, i.e., 1, 1, 1.

$\therefore$ The required plane passes through N(4, 5, 6) and is normal to AB.

Thus its equation is

$1(x-4) + 1(y-5) + 1(z-6) = 0 \Rightarrow x + y + z = 15.$



48. (b) The direction ratios of the line are $3 - 2, -4 - (-3), -5 - 1$, i.e., $1, -1, -6$

Hence, equation of the line joining the given points is $\frac{x-2}{1} = \frac{y+3}{-1} = \frac{z-1}{-6} = r$(say)

Coordinates of any point on this line are $(r + 2, -r - 3, -6r + 1)$

If this point lies on the given plane $2x + y + z = 7$, then $2(r + 2) + (-r - 3) + (-6r + 1) = 7 \Rightarrow r = -1$

Coordinates of any point on this line are $(-1 + 2, -(-1) - 3, -6(-1) + 1)$, i.e., $(1, -2, 7)$.

50. (c) The point $(4, 2, k)$ on the line also lies on the plane $2x - 4y + z = 7$.

So, $8 - 8 + k = 7 \Rightarrow k = 7$

51. (a) The given lines are :

$$\frac{x-2}{1} = \frac{y-(-1)}{-2} = \frac{z-(-2)}{1} \text{ and } \frac{x-1}{1} = \frac{y-\left(-\frac{3}{2}\right)}{\frac{3}{2}} = \frac{z-(-5)}{2}$$

D.R.'S of Ist line are: $a_1 = 1, b_1 = -2, c_1 = 1$

D.R.'S of IInd line are: $a_2 = 2, b_2 = 3, c_2 = 4$

Let 'θ' be the angle between two lines, then

$$\cos\theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{a_1^2+b_1^2+c_1^2}\cdot\sqrt{a_2^2+b_2^2+c_2^2}}$$

$\cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}.$

53. (c) We have, $z = 0$ for the point where the line intersects the curve. Therefore,

$$\frac{x-2}{3} = \frac{y+1}{2} = \frac{0-1}{-1} \Rightarrow \frac{x-2}{3} = 1 \text{ and } \frac{y+1}{2} = 1 \Rightarrow x = 5 \text{ and } y = 1$$

Put these value in $xy = c^2$, we get, $5 = c^2$

$\Rightarrow c = \pm\sqrt{5}.$

54. (a) Direction ratios of AB are $(4, -4, -2) = (2, -2, -1)$ $a^2 + b^2 + c^2 = 9$

Direction cosines are $\left(\pm\frac{2}{3}, \pm\frac{2}{3}, \pm\frac{1}{3}\right)$

55. (c) Let the components of the line vector be a, b, c. Then, $a^2 + b^2 + c^2 = (63)^2$

Also $\frac{a}{3} = \frac{b}{-2} = \frac{c}{6} = \lambda$ (say), then $a = 3\lambda$, $b = -2\lambda$ and $c = 6\lambda$ and from (i) we have

$9\lambda^2 + 4\lambda^2 + 36\lambda^2 = (63)^2 \Rightarrow 49\lambda^2 = (63)^2 \Rightarrow \lambda = \pm\frac{63}{7} = \pm 9$

Since, $a = 3\lambda < 0$ as the line makes an obtuse angle with x-axis, $\lambda = -9$ and the required components are $-27, 18, -54$.

56. (b) Let yz-plane divides the line segment joining A and B in the ratios $\lambda : 1$. The coordinates of the point C of division are $\left(\frac{3\lambda+5}{\lambda+1}, \frac{4\lambda+1}{\lambda+1}, \frac{\lambda+6}{\lambda+1}\right)$

This point lies on yz-plane. $\frac{3\lambda+5}{\lambda+1} = 0 \Rightarrow \lambda = \frac{-5}{3}.$

Hence, the coordinates of c are $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$.

60. (c) Let the length of the line segment be r its direction cosines be l, m, n, . Then its projections on the coordinate axes are lr, mr, nr.

$\therefore lr = 2, mr = 3$ and $nr = 6$

$\Rightarrow l^2r^2 + m^2r^2 + n^2r^2 = 4 + 9 + 36 \Rightarrow r^2 = 49 \Rightarrow r = 7.$

61. (c) Let l, m, n be the direction cosine

Then, $l = \cos\theta$, $m = \cos\beta$ and $n = \cos\theta$

$\therefore l^2 + m^2 + n^2 = 1 \Rightarrow \cos^2\theta + \cos^2\beta + \cos^2\theta = 1$

$\Rightarrow 2\cos^2\theta + 1 - \sin^2\beta = 1 \Rightarrow 2\cos^2\theta - 3\sin^2\theta = 0$

$\Rightarrow 5\cos^2\theta = 3 \Rightarrow \cos^2\theta = \frac{3}{5}.$

62. (a) We have, $l = \cos 45° = \frac{1}{\sqrt{2}}$, $m = \cos 120° = \frac{-1}{2}$ and $n = \cos\theta$

$\Rightarrow l^2 + m^2 + n^2 = 1 \Rightarrow \frac{1}{2} + \frac{1}{4} + \cos^2\theta = 1 \Rightarrow \cos^2\theta = \frac{1}{4}$

$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°.$

63. (b) Vectors normal to the given planes are $\vec{n}_1 = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{n}_2 = 3\hat{i} + \hat{j} + \lambda\hat{k}$

If the planes are perpendicular, then $\vec{n}_1 \cdot \vec{n}_2 = 0 \Rightarrow 6 - 2 + 2\lambda = 0 \Rightarrow \lambda = -2$

65. (d) Here, $\frac{x}{3} = \frac{y}{2} = \frac{z}{-6}$ and $\frac{x}{1} = \frac{y}{-6} = \frac{z}{-3/2}$

Clearly, $3 \times 1 + 2 \times -6 - 6 \times \frac{-3}{2} = 0$

So, given lines are perpendicular to each other.

66. (d) We have, the line of intersection of the planes

$\vec{r} \cdot \vec{n}_1 = d_1$ and $\vec{r} \cdot \vec{n}_2 = d_2$ is parallel to $\vec{n}_1 \times \vec{n}_2$.

Here, $\vec{n}_1 = 3\hat{i} - \hat{j} + \hat{k}$ and $\vec{n}_2 = \hat{i} + 4\hat{j} - 2\hat{k}$

$$\therefore \text{Required vector} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ 1 & 4 & -2 \end{vmatrix} = -2\hat{i} - 7\hat{j} + 13\hat{k}.$$