Exercise 6.1 Page: 78

1. Fill in the blanks using the correct word given in brackets:

(i) All circles are ________________. (congruent, similar)

(ii) All squares are ________________. (similar, congruent)

(iii) All ________________ triangles are similar. (isosceles, equilateral)

(iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are ________________ and (b) their corresponding sides are ________________. (equal, proportional)

Answer:

(i) similar

(ii) similar

(iii) equilateral

(iv) equal, proportional

2. Give two different examples of pair of:

(i) similar figures

(ii) non-similar figures

Answer:

(i) Two different examples of pair of similar figures are:

(a) Any two rectangles

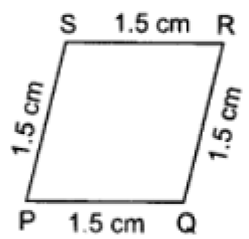
(b) Any two squares

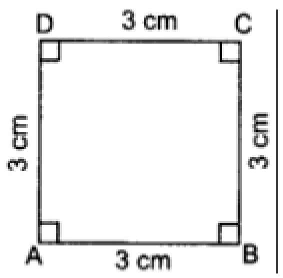
(ii) Two different examples of pair of non-similar figures are:

(a) A scalene and an equilateral triangle

(b) An equilateral triangle and a right angled triangle

3. State whether the following quadrilaterals are similar or not:



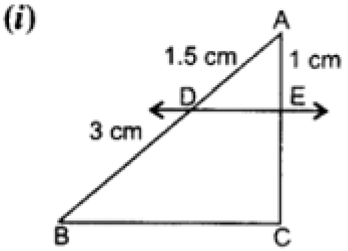


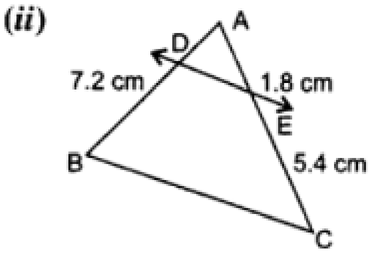
Answer:

On looking at the given figures of the quadrilaterals, we can say that they are not similar because their angles are not equal.

Exercise 6.2 Page: 84

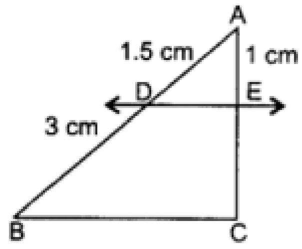
1. In figure (i) and (ii), DE || BC. Find EC in (i) and AD in (ii).





Answer:

(i) Since DE || BC,



Let, EC = x cm

It is given that DE || BC

By using basic proportionality theorem, we obtain

$$\frac{AD}{DB} = \frac{AE}{EC}$$

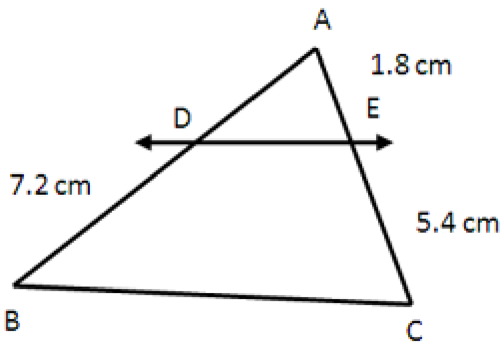
$$\frac{1.5}{3} = \frac{1}{x}$$

$$x = \frac{3 \times 1}{1.5}$$

x = 2

EC = 2 cm.

(ii) Let AD = x cm.



It is given that DE || BC.

By using basic proportionality theorem, we obtain

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x}{7.2} = \frac{1.8}{5.4}$$

$$x = \frac{1.8 \times 7.2}{5.4}$$

x = 2.4

AD = 2.4cm.

2. E and F are points on the sides PQ and PR respectively of a Δ PQR. For each of the following cases, state whether EF ll QR:

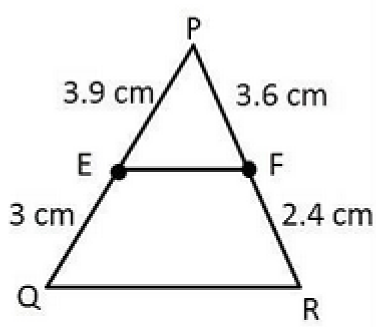
(i) PE = 3.9 cm, EQ = 3cm, PF = 3.6 cm and FR = 2.4 cm

(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm

(iii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm

Answer:

(i)Given: PE = 3.9 cm, EQ = 3cm, PF = 3.6 cm and FR = 2.4 cm



$$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$$

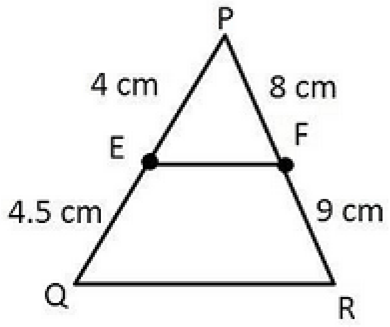
$$\frac{PF}{FR} = \frac{3.6}{2.4} = 1.5$$

Hence,

$$\frac{PE}{EQ} \neq \frac{PF}{FR}$$

Therefore, EF is not parallel to QR.

(ii)Given: PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm



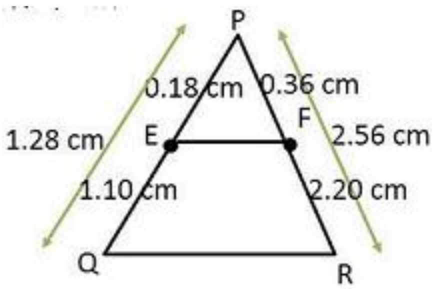
$$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$$

$$\frac{PF}{FR} = \frac{8}{9}$$

$$\frac{PF}{FR} = \frac{PE}{EQ}$$

Therefore ,EF is parallel to QR.

(iii)Given: PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm



And ER = PR – PF = 2.56 – 0.36 = 2.20 cm

$$\frac{PE}{PQ} = \frac{0.18}{1.28} = \frac{18}{128} = \frac{9}{64}$$

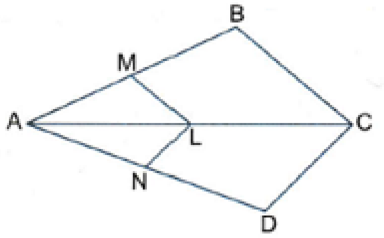
$$\frac{PF}{PR} = \frac{0.36}{2.56} = \frac{9}{64}$$

Hence,

$$\frac{PE}{PQ} = \frac{PF}{PR}$$

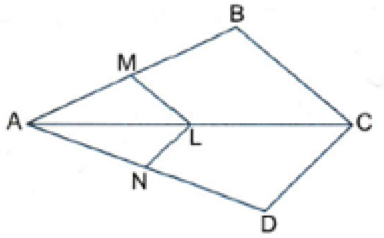
Therefore, EF is parallel to QR.

3. In figure, if LM || CB and LN || CD, prove that AM/AB = AN/AD.



Answer:

In figure, LM || CB



By using Basic Proportionality theorem

And in Δ ACD, LN || CD

$$\frac{AM}{AB} = \frac{AL}{AC}$$ i

Similalry, in Δ ACD, LN || CD

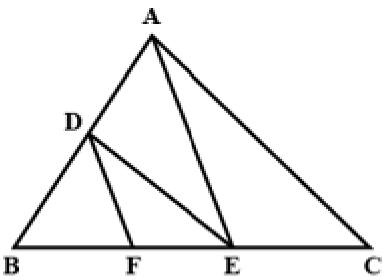
$\therefore \frac{AN}{AD} = \frac{AL}{AC}$ ii

By using Basic Proportionality theorem

From eq. (i) and (ii), we have

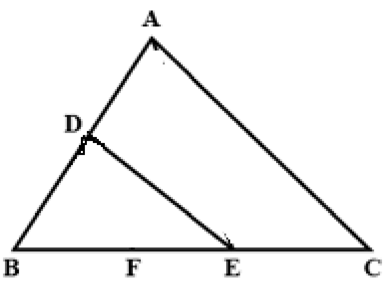
$\frac{AM}{AD} = \frac{AN}{AD}$

4. In figure, DE || AC and DF || AE. Prove that BF/GE = BE/EC .

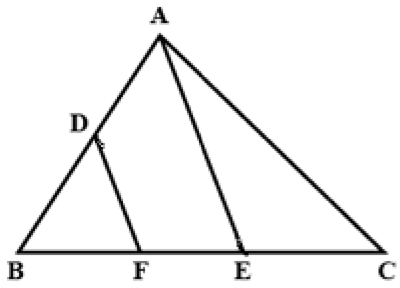


Answer:

In Δ BCA, DE || AC



$\frac{BD}{DA} = \frac{BE}{EC}$ [Basic Proportionality theorem] ……….(i)



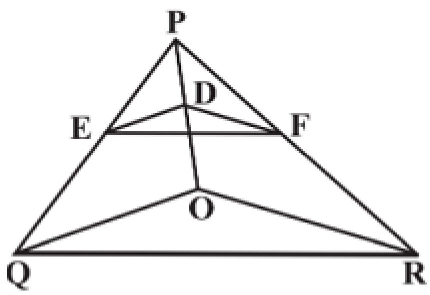
And in ∆ BEA, DF || AE

$\frac{BD}{DA} = \frac{BF}{FE}$ [Basic Proportionality theorem] ……….(ii)

From eq. (i) and (ii), we have

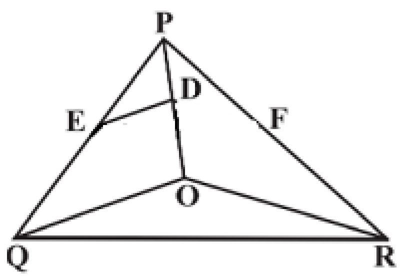
$\frac{BE}{EC} = \frac{BF}{FE}$

5. In figure, DE || OQ and DF || OR. Show that EF || QR.



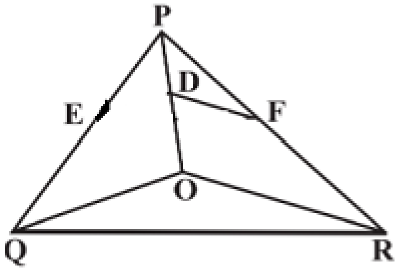
Answer:

In ∆ PQO, DE || OQ



$\frac{PE}{EQ} = \frac{PD}{DO}$ [Basic Proportionality theorem] ……….(i)

And in Δ POR, DF || OR

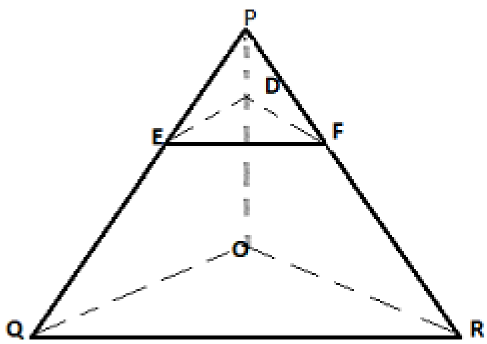


$\frac{PF}{FR} = \frac{PD}{DO}$ [Basic Proportionality theorem] ……….(ii)

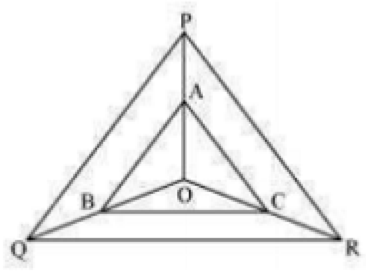
From eq. (i) and (ii), we have

$\frac{PE}{EQ} = \frac{PF}{FR}$

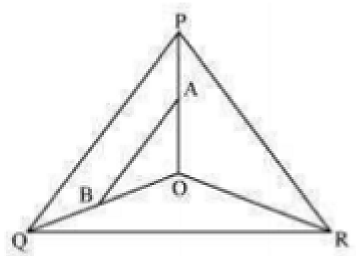
Therefore, EF || QR [By the converse of Basic Proportionality Theorem]



6. In figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.

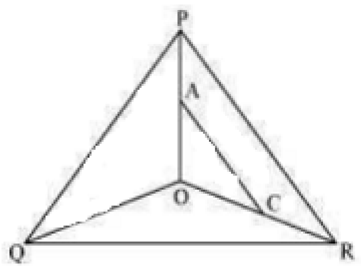


Answer:



And in Δ POQ, AB || PQ

$\frac{OA}{AP}=\frac{OB}{BQ}$ [Basic Proportionality theorem] ……….(i)



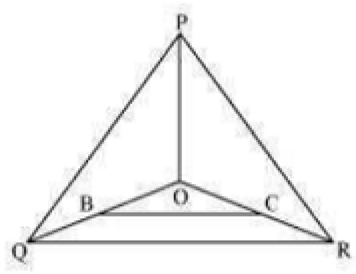
And in Δ OPR, AC || PR

$\frac{OA}{AP}=\frac{OC}{CR}$ [Basic Proportionality theorem] ……….(ii)

From eq. (i) and (ii), we have

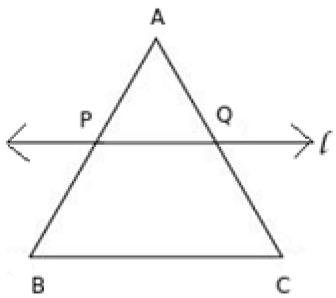
$\frac{OB}{BQ}=\frac{OC}{CR}$

Therefore, BC || QR (By the converse of Basic Proportionality Theorem)



7. Using Theorem 6.1, prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

Answer:



Consider the given figure in which l is a line drawn through the mid-point P of line segment AB meeting AC at Q, such that PQ || BC

By using Basic Proportionality theorem, we obtain,

$$\frac{AQ}{QC} = \frac{AP}{PB}$$

$\frac{AQ}{QC} = \frac{1}{1}$ (P is the midpoint of AB ∴ AP = PB)

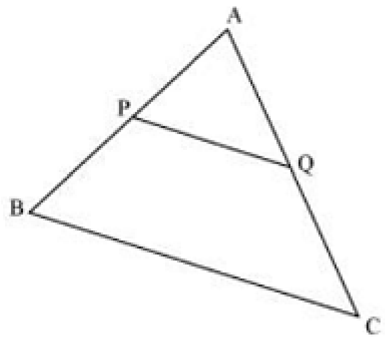
⇒ AQ = QC

Or, Q is the mid-point of AC.

8. Using Theorem 6.2, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Answer:

Given: A triangle ABC, in which P and Q are the mid-points of sides AB and AC respectively.



Consider the given figure in which PQ is a line segment joining the mid-points P and Q of line AB and AC respectively.

i.e., AP = PB and AQ = QC

It can be observed that

$$\frac{AP}{PB} = \frac{1}{1}$$

and $\frac{AQ}{QC} = \frac{1}{1}$

Therefore, $\frac{AP}{PB} = \frac{AQ}{QC}$

Hence, by using basic proportionality theorem, we obtain

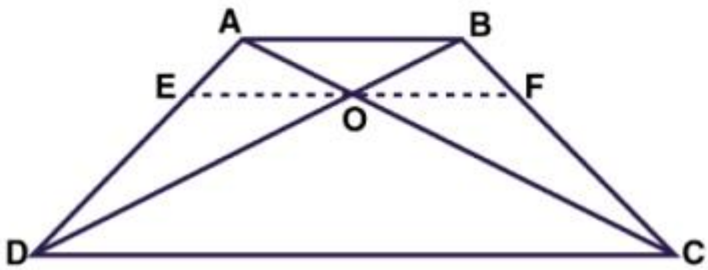
PQ || BC.

9. ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that AO/BO = CO/DO .

Answer:

Given: A trapezium ABCD, in which AB || DC and its diagonals

AC and BD intersect each other at O.



Draw a line EF through point O, such that EF || CD

In ΔADC, EO || CD

By using basic proportionality theorem, we obtain

$\frac{AE}{ED} = \frac{AO}{OC}$ ...i

In ΔABD, OE || AB

So, by using basic proportionality theorem, we obtain

$\frac{ED}{AE} = \frac{OD}{BO}$

$\Rightarrow \frac{AE}{ED} = \frac{BO}{OD}$ ...ii

From eq. (i) and (ii), we get

$$\frac{AO}{OC} = \frac{BO}{OD}$$

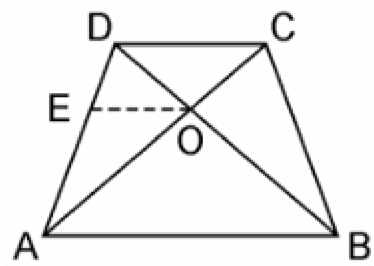
$$\Rightarrow \frac{AO}{BO} = \frac{OC}{OD}$$

10. The diagonals of a quadrilateral ABCD intersect each other at the point O such that AO/BO = CO/DO Such that ABCD is a trapezium.

Answer:

Given: A quadrilateral ABCD, in which its diagonals AC and

BD intersect each other at O such that , i.e.



Quadrilateral ABCD is a trapezium.

Construction: Through O, draw OE || AB meeting AD at E.

In Δ ADB, we have OE || AB [By construction] By Basic Proportionality theorem

$$\frac{AE}{ED} = \frac{BO}{OD} \quad .....i$$

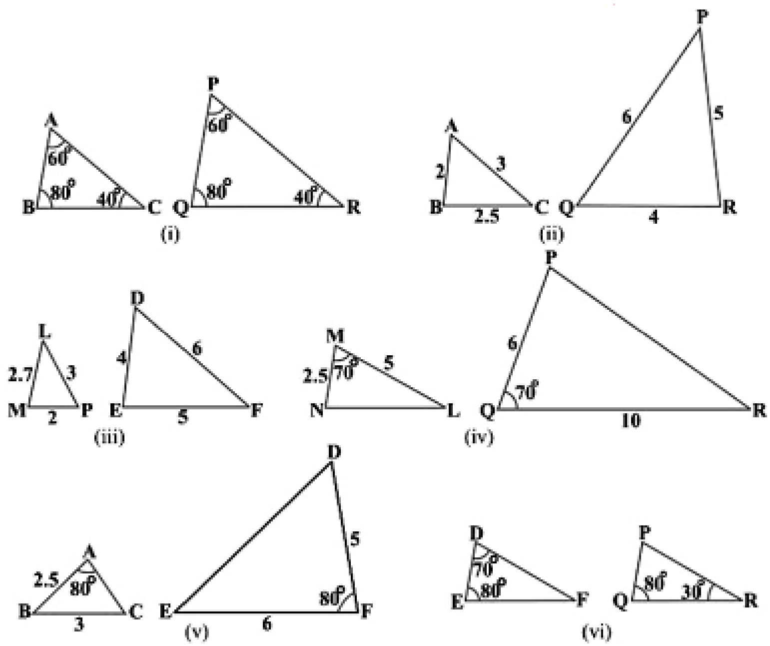
However, it is given that

$$\frac{AO}{OC} = \frac{OB}{OD} \quad .....ii$$

From eq. (i) and (ii), we get

Exercise 6.3 Page: 94

1. State which pairs of triangles in figure, are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:



Answer:

(i) In ΔABC and ΔPQR, we have
∠A = ∠P = 60° (Given)
∠B = ∠Q = 80° (Given)
∠C = ∠R = 40° (Given)
∴ ΔABC ~ ΔPQR (AAA similarity criterion)

(ii) In ΔABC and ΔPQR, we have
AB/QR = BC/RP = CA/PQ
∴ ΔABC ~ ΔQRP (SSS similarity criterion)

(iii) In ΔLMP and ΔDEF, we have
LM = 2.7, MP = 2, LP = 3, EF = 5, DE = 4, DF = 6
MP/DE = 2/4 = 1/2

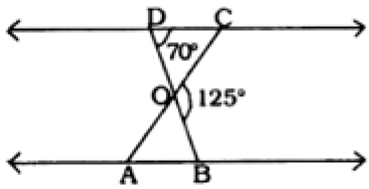
PL/DF = 3/6 = 1/2
LM/EF= 2.7/5 = 27/50
Here, MP/DE = PL/DF ≠ LM/EF
Hence, ΔLMP and ΔDEF are not similar.

(iv) In ΔMNL and ΔQPR, we have
MN/QP = ML/QR = 1/2
∠M = ∠Q = 70°
∴ ΔMNL ~ ΔQPR (SAS similarity criterion)

(v) In ΔABC and ΔDEF, we have
AB = 2.5, BC = 3, ∠A = 80°, EF = 6, DF = 5, ∠F = 80°
Here, AB/DF = 2.5/5 = 1/2
And, BC/EF = 3/6 = 1/2
⇒ ∠B ≠ ∠F
Hence, ΔABC and ΔDEF are not similar.

(vi) In ΔDEF,we have
∠D + ∠E + ∠F = 180° (sum of angles of a triangle)
⇒ 70° + 80° + ∠F = 180°
⇒ ∠F = 180° - 70° - 80°
⇒ ∠F = 30°
In PQR, we have
∠P + ∠Q + ∠R = 180 (Sum of angles of Δ)
⇒ ∠P + 80° + 30° = 180°
⇒ ∠P = 180° - 80° -30°
⇒ ∠P = 70°
In ΔDEF and ΔPQR, we have
∠D = ∠P = 70°
∠F = ∠Q = 80°
∠F = ∠R = 30°
Hence, ΔDEF ~ ΔPQR (AAA similarity criterion)

2. In the figure, ΔODC ∝ ¼ ΔOBA, ∠ BOC = 125° and ∠ CDO = 70°. Find ∠ DOC, ∠ DCO and ∠ OAB.



Answer:

DOB is a straight line.

∴ ∠DOC + ∠COB = 180°

⇒ ∠DOC = 180° − 125°

= 55°

In ΔDOC,

∠DCO + ∠CDO + ∠DOC = 180°

(Sum of the measures of the angles of a triangle is 180º.)

⇒ ∠DCO + 70º + 55º = 180°

⇒ ∠DCO = 55°

It is given that ΔODC ~ ΔOBA.

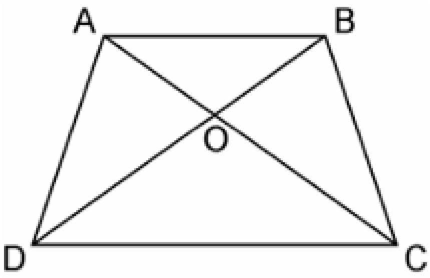
∴ ∠OAB = ∠ OCD [Corresponding angles are equal in similar triangles.]

⇒ ∠OAB = 55°.

3. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles, show that AO/OC = OB/OD

Answer:

Given: ABCD is a trapezium in which AB DC.



In ΔDOC and ΔBOA,

∠CDO = ∠ABO [Alternate interior angles as AB || CD]

∠DCO = ∠BAO [Alternate interior angles as AB || CD]

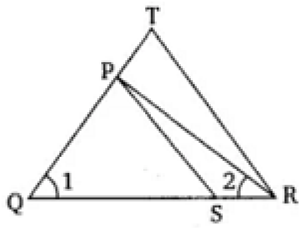
∠DOC = ∠BOA [Vertically opposite angles]

∴ ΔDOC ~ ΔBOA [AAA similarity criterion]

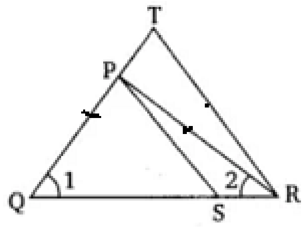
∴ $\frac{DO}{BO} = \frac{OC}{OA}$ ..... [Corresponding sides are proportional]

Hence, $\frac{OA}{OC} = \frac{OB}{OD}$

4. In the figure, QR/QS = QT/PR and ∠1 = ∠2. Show that ΔPQS ~ ΔTQR.



Answer:

We have, 

In ΔPQR, ∠PQR = ∠PRQ

∴ PQ = PR .....(i)

Given,

$$\frac{QR}{QS} = \frac{QT}{PR}$$

Using (i), we get,

$$\frac{QR}{QS} = \frac{QT}{QP}$$

In ΔPQS & In ΔTQR

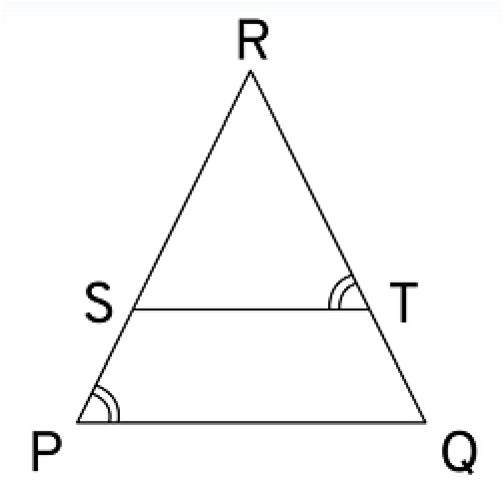
$$\frac{QR}{QS} = \frac{QT}{QP}$$

∠Q = ∠Q

∴ ΔPQS ~ ΔTQR [SAS similarity criterion]

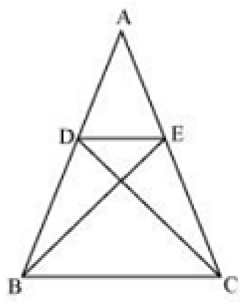
5. S and T are point on sides PR and QR of ΔPQR such that ∠P = ∠RTS. Show that ΔRPQ ~ ΔRTS.

Answer:



In ΔRPQ and ΔRST,
∠RTS = ∠QPS (Given)
∠R = ∠R (Common angle)
∴ ΔRPQ ~ ΔRTS (By AA similarity criterion)

6. In the figure, if ΔABE ≅ ΔACD, show that ΔADE ~ ΔABC.

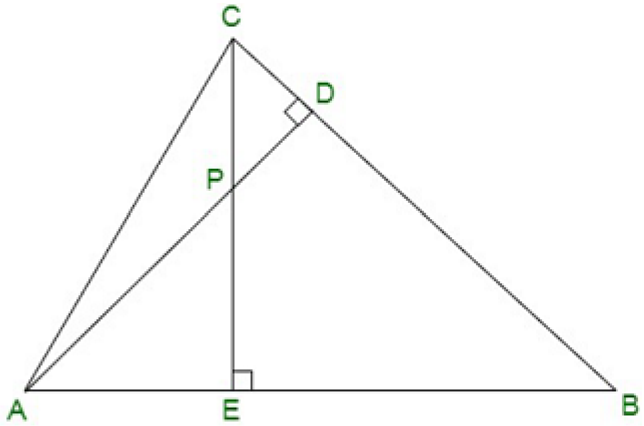


Answer:

It is given that ΔABE ≅ ΔACD.
∴ AB = AC [By cpct] ...(i)
And, AD = AE [By cpct] ...(ii)
In ΔADE and ΔABC,
AD/AB = AE/AC [Dividing equation (ii) by (i)]
∠A = ∠A [Common angle]
∴ ΔADE ~ ΔABC [By SAS similarity criterion]

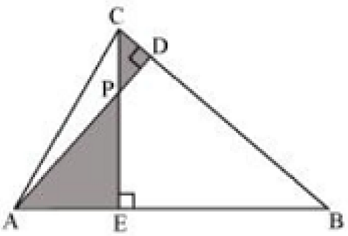
7. In the figure, altitudes AD and CE of ΔABC intersect each other at the point P. Show that:

(i) ΔAEP ~ ΔCDP
(ii) ΔABD ~ ΔCBE
(iii) ΔAEP ~ ΔADB
(iv) ΔPDC ~ ΔBEC



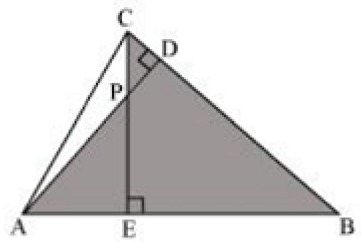
Answer:

(i) In ΔAEP and ΔCDP,



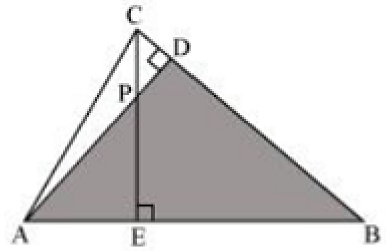
∠AEP = ∠CDP (Each 90°)
∠APE = ∠CPD (Vertically opposite angles)
Hence, by using AA similarity criterion,
ΔAEP ~ ΔCDP

(ii) In ΔABD and ΔCBE,



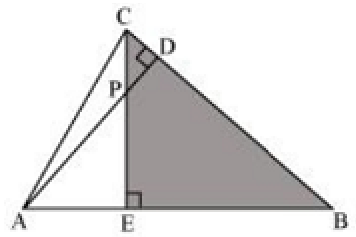
∠ADB = ∠CEB (Each 90°)
∠ABD = ∠CBE (Common)
Hence, by using AA similarity criterion,
ΔABD ~ ΔCBE

(iii) In ΔAEP and ΔADB,



∠AEP = ∠ADB (Each 90°)
∠PAE = ∠DAB (Common)
Hence, by using AA similarity criterion,
ΔAEP ~ ΔADB

(iv) In ΔPDC and ΔBEC,

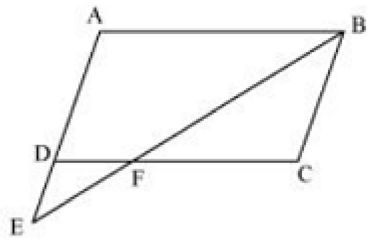


∠PDC = ∠BEC (Each 90°)
∠PCD = ∠BCE (Common angle)
Hence, by using AA similarity criterion,
ΔPDC ~ ΔBEC

8. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that ΔABE ~ ΔCFB.

Answer:

In ΔABE and ΔCFB,we have,

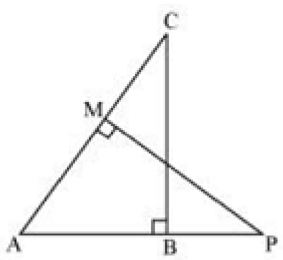


∠A = ∠C (Opposite angles of a parallelogram)
∠AEB = ∠CBF (Alternate interior angles as AE || BC)

∴By AA-criterion of similarity, we have

∴ ΔABE ~ ΔCFB (By AA similarity criterion)

9. In the figure, ABC and AMP are two right triangles, right angled at B and M respectively, prove that:
(i) ΔABC ~ ΔAMP
(ii) CA/PA = BC/MP



Answer:

(i) In Δ ABC and AMP, we have,

∠ABC = ∠AMP =90^0 [Given]

∠BAC = ∠MAP [Common angles]

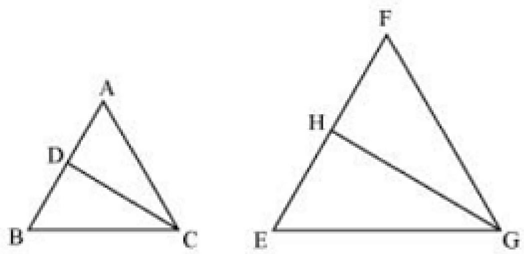
∴ Δ ABC ~ Δ AMP [By AA-criterion of similarity, we have]

⇒ CA/PA = BC/MP ..... (Corresponding sides of similar trianlges are proportional)

10. CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively. If ΔABC ~ ΔFEG, Show that:
(i) CD/GH = AC/FG
(ii) ΔDCB ~ ΔHGE
(iii) ΔDCA ~ ΔHGF

Answer:

We have, Δ ABC ~ Δ FEG



∴ ∠A = ∠F, ∠B = ∠E, & ∠ACB = ∠FGE

∴ ∠ACD = ∠FGH (Angle Bisector)

And ∠DCB = ∠HGE (Angle Bisector)

In ΔACD & ΔFGH

∠A = ∠F (Proved above)

∠ACD = ∠FGH (Proved above)

∴ Δ ACD ~ Δ FGH [By AA similarity criterion]

⇒ [(CD)/ (GH)] = [(AC) / (FG)]

In ΔDCB & ΔHGE,

∠DCB= ∠HGE (Proved above)

∠B= ∠E (Proved above)

∴ Δ DCB~ Δ HGE [By AA similarity criterion]

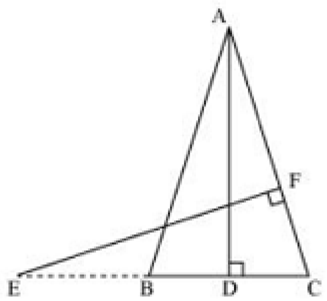
In ΔDCA & ΔHGF,

∠ACD = ∠FGH (Proved above)

∠A = ∠F (Proved above)

∴ Δ DCA~ Δ HGF [By AA similarity criterion]

11. In the following figure, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and EF ⊥ AC, prove that ΔABD ~ ΔECF.



Answer:

Here Δ ABC is isosceles with AB = AC

∠B = ∠C

In Δ ABD and ECF, we have

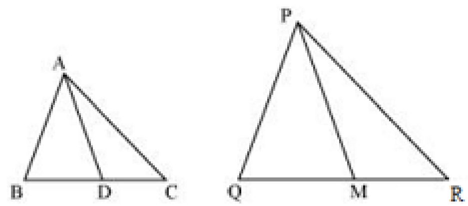
∠ABD = ∠ECF[Each 90°]

∠ABD = ∠ECF = [Proved above]

By AA-criterion of similarity, we have

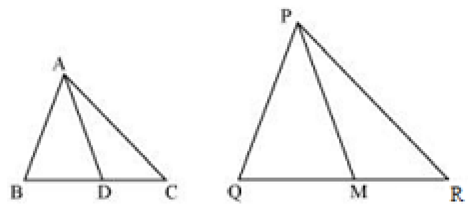
Δ ABD ~ Δ ECF

12. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR (see figure). Show that ΔABC ~ ΔPQR.



Answer:

Given: AD is the median of Δ ABC and PM is the median of Δ PQR such that



Median divides the opposite side.

∴ BD = BC / 2 [Given]

And QM = QR / 2 [Given]

Given that,

AB/PQ = BC/QR = AD/PM

⇒ AB/PQ =[(½BC) / (½QR)]= AD/PM

⇒ AB/PQ = BD/QM = AD/PM

In ΔABD and ΔPQM,

AB/PQ = BD/QM = AD/PM [Proved above]

∴ ΔABD ~ ΔPQM (By SSS similarity criterion)

⇒ ∠ABD = ∠PQM (Corresponding angles of similar triangles)

In ΔABC and ΔPQR,

∠ABD = ∠PQM (Proved above)

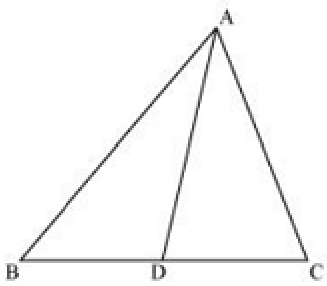
AB / PQ = BC / QR

∴ ΔABC ~ ΔPQR (By SAS similarity criterion)

13. D is a point on the side BC of a triangle ABC such that ∠ADC = ∠BAC. Show that CA^2 = CB.CD

Answer:

In triangles ABC and DAC,

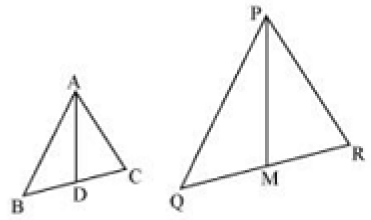


∠ADC = ∠BAC (Given)
∠ACD = ∠BCA (Common angle)
∴ ΔADC ~ ΔBAC (By AA similarity criterion)
We know that corresponding sides of similar triangles are in proportion.
∴ CA/CB =CD/CA
⇒ CA^2 = CB x CD.

14. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that ΔABC ~ ΔPQR.

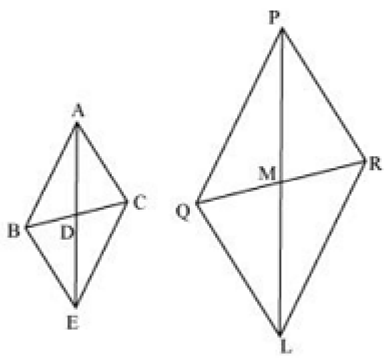
Answer:

Given: AD is the median of Δ ABC and PM is the median of Δ PQR such that



⇒AB / PQ = AC / PR = AD / PM

Let us extend AD and PM up to point E and L respectively, such that AD = DE and PM = ML. Then, join B to E, C to E, Q to L, and R to L.



We know that medians divide opposite sides.

Therefore, BD = DC and QM = MR

Also, AD = DE (By construction)

And, PM = ML (By construction)

In quadrilateral ABEC, diagonals AE and BC bisect each other at point D.

Therefore, quadrilateral ABEC is a parallelogram.

∴ AC = BE and AB = EC (Opposite sides of a parallelogram are equal)

Similarly, we can prove that quadrilateral PQLR is a parallelogram and PR = QL, PQ = LR

It was given that

⇒AB / PQ = AC / PR = AD / PM

⇒AB / PQ = BE / QL = [(2AD) / (2PM)]

⇒AB / PQ = BE / QL = AE / PL

∴ ΔABE ~ ΔPQL (By SSS similarity criterion)

We know that corresponding angles of similar triangles are equal.

∴ ∠BAE = ∠QPL … (1)

Similarly, it can be proved that ΔAEC ~ ΔPLR and

∠CAE = ∠RPL … (2)

Adding equation (1) and (2), we obtain

∠BAE + ∠CAE = ∠QPL + ∠RPL

⇒ ∠CAB = ∠RPQ … (3)

In ΔABC and ΔPQR,

AB / PQ = AC / PR

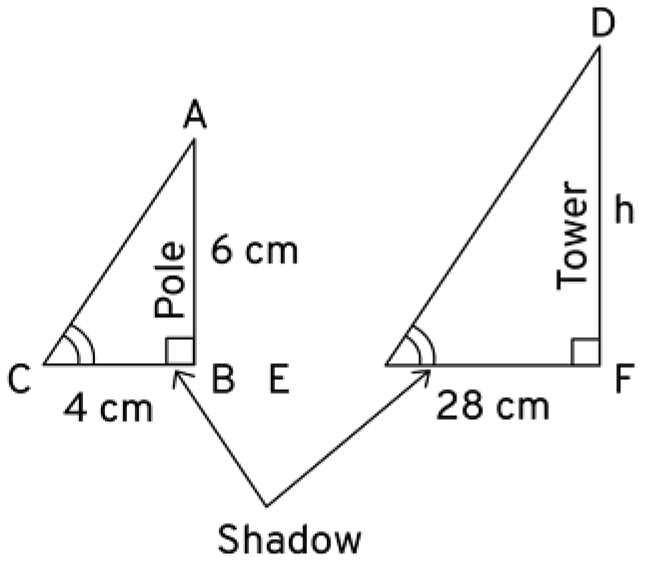
∠CAB = ∠RPQ [Using equation (3)]

∴ ΔABC ~ ΔPQR (By SAS similarity criterion)

15. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Answer:

Let AB the vertical pole and AC be its shadow. Also, let DE be the vertical tower and DF be its shadow. Joined BC and EF.



Length of the vertical pole = 6m (Given)

Shadow of the pole = 4 m (Given)

Let Height of tower = h m

Length of shadow of the tower = 28 m (Given)

In ΔABC and ΔDEF,

∠C = ∠E (angular elevation of sum)

∠B = ∠F = 90°

∴ ΔABC ~ ΔDEF (By AA similarity criterion)

∴ AB/DF = BC/EF (If two triangles are similar corresponding sides are proportional)

∴ 6/h = 4/28

⇒ h = 6 x 28/4

⇒ h = 6 x 7

⇒ h = 42 m

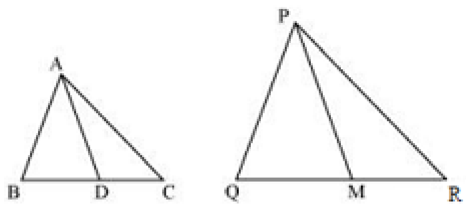
Hence, the height of the tower is 42 meters.

16. If AD and PM are medians of triangles ABC and PQR, respectively where ΔABC ~ ΔPQR prove that AB/PQ = AD/PM.

Answer:

Given: AD and PM are the medians of triangles

ABC and PQR respectively, where



It is given that Δ ABC ~ Δ PQR

We know that the corresponding sides of similar triangles are in proportion.

∴ AB / PQ = AC / AD and BC / QR ....(1)

Also, ∠A = ∠P, ∠B = ∠Q, ∠C = ∠R … (2)

Since AD and PM are medians, they will divide their opposite sides.

∴ BD = BC/2 & QM = QR/2 ...(3)

From equations (1) and (3), we obtain

AB/PQ = BD/QM ... (4)

In ΔABD and ΔPQM,

∠B = ∠Q [Using equation (2)]

AB/PQ = BD/QM

∴ ΔABD ~ ΔPQM (By SAS similarity criterion)

AB/PQ = BD/QM = AD/PM

$$\frac{AE}{ED} = \frac{AO}{OC}$$

⇒ EO || DC [By the converse of basic proportionality theorem]

⇒ AB || OE || DC

⇒ AB || CD

∴ ABCD is a trapezium.