CHAPTER 1 NUMBER SYSTEMS

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Question 1. Write the following in decimal form and say what kind of decimal expansion each has: $\sqrt{5}$

(i) $\frac{36}{100}$

(ii) $\frac{1}{11}$

(iii) $4\frac{1}{8}$

(iv) $\frac{3}{13}$

(v) $\frac{2}{11}$

(vi) $\frac{329}{400}$

Solution :

(i) $\frac{36}{100}$

On dividing 36 by 100, we get

$$
\begin{array}{r}
0.36 \\
100\overline{)\ 36} \\
\underline{-0} \\
360 \\
\underline{-300} \\
600 \\
\underline{-600} \\
\underline{0}
\end{array}
$$

Therefore, we conclude that $\frac{36}{100} = 0.36$, which is a terminating decimal.

(ii) $\frac{1}{11}$

On dividing 1 by 11, we get

$$
\begin{array}{r}
0.0909.... \\
11\overline{)\qquad 1} \\
\underline{-0} \\
10 \\
\underline{-0} \\
100 \\
\underline{-99} \\
10 \\
\underline{-0} \\
100 \\
\underline{-99} \\
\underline{1}
\end{array}
$$

We can observe that while dividing 1 by 11, we got the remainder as 1, which will continue to be 1.

Therefore, we conclude that $4\frac{1}{8} = \frac{33}{8} = 4.125$, which is a non-terminating decimal and recurring decimal.

(iii) $4\frac{1}{8} = \frac{33}{8}$

On dividing 33 by 8, we get

$$
\begin{array}{r}
4.125 \\
8\overline{)\;33} \\
\underline{-32} \\
10 \\
\underline{-8} \\
20 \\
\underline{-16} \\
40 \\
\underline{-40} \\
\underline{0}
\end{array}
$$

We can observe that while dividing 33 by 8, we got the remainder as 0.

Therefore, we conclude that $4\frac{1}{8}=\frac{33}{8}=4.125$, which is a terminating decimal.

(iv) $\frac{3}{13}$

On dividing 3 by 13, we get

$$
\begin{array}{r}
0.230769.... \\
13\overline{)3} \\
\underline{-0} \\
30 \\
\underline{-26} \\
40 \\
\underline{-39} \\
10 \\
\underline{-0} \\
100 \\
\underline{-91} \\
90 \\
\underline{-78} \\
120 \\
\underline{-117} \\
\underline{3}
\end{array}
$$

We can observe that while dividing 3 by 13 we got the remainder as 3, which will continue to be 3 after carrying out 6 continuous divisions.

Therefore, we conclude that $\frac{3}{13} = 0.230769.....$ or $\frac{3}{13} = 0.\overline{230769}$, which is a non-terminating decimal and recurring decimal.

(v) $\frac{2}{11}$

On dividing 2 by 11, we get

$$
\begin{array}{r|l}
 & 0.1818.... \\
11 & \quad 2 \\
 & \underline{-0} \\
 & \quad 20 \\
 & \underline{-11} \\
 & \quad 90 \\
 & \underline{-88} \\
 & \quad 20 \\
 & \underline{-11} \\
 & \quad 90 \\
 & \underline{-88} \\
 & \quad \underline{2}
\end{array}
$$

We can observe that while dividing 2 by 11, first we got the remainder as 2 and then 9, which will continue to be 2 and 9 alternately.

Therefore, we conclude that $\frac{2}{11} = 0.1818.....$ or $\frac{2}{11} = 0\overline{18}$, which is a non-terminating decimal and recurring decimal.

(vi) $\frac{329}{400}$

On dividing 329 by 400, we get

$$\begin{array}{r} 0.8225 \\ 400\overline{)\ 329} \\ \underline{-0} \\ 3290 \\ \underline{-3200} \\ 900 \\ \underline{-800} \\ 1000 \\ \underline{-800} \\ 2000 \\ \underline{-2000} \\ \underline{0} \end{array}$$

We can observe that while dividing 329 by 400, we got the remainder as 0.

Therefore, we conclude that $\frac{329}{400} = 0.8225$, which is a terminating decimal.

Question 2.

You know that $\frac{1}{7} = 0.142857.....$. Can you predict what the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are, without actually doing the long division? If so, how?

[Hint: Study the remainders while finding the value of $\frac{1}{7}$ carefully.]

Solution :

We are given that $\frac{1}{7} = 0.\overline{142857}$ or $\frac{1}{7} = 0.142857.....$.

We need to find the values of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}$ and $\frac{6}{7}$, without performing long division.

We know that, $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}$ and $\frac{6}{7}$ can be rewritten as

$2 \times \frac{1}{7}, 3 \times \frac{1}{7}, 4 \times \frac{1}{7}, 5 \times \frac{1}{7}$ and $6 \times \frac{1}{7}$.

On substituting value of $\frac{1}{7}$ as $0.142857.....$, we get

$5\times\frac{1}{7}=5\times0.142857.....=0.285714.........$

$3\times\frac{1}{7}=3\times0.142857....=0.428571$

$4\times\frac{1}{7}=4\times0.142857...=0.571428$

$5\times\frac{1}{7}=5\times0.142857.....=0.714285$

$6\times\frac{1}{7}=6\times0.142857.....=0.857142$

Therefore, we conclude that, we can predict the values of $\frac{2}{7},\frac{3}{7},\frac{4}{7},\frac{5}{7}$ and $\frac{6}{7}$, without performing long division, to get

$\frac{2}{7}=0.\overline{285714},\frac{3}{7}=0.\overline{428571},\frac{4}{7}=0.\overline{571428},\frac{5}{7}=0.\overline{714285},$ and $\frac{6}{7}=0.\overline{857142}$

Question 3. Express the following in the form $\frac{p}{q}$, where p and q are integers and q $\neq$ 0.

(i) $0.\overline{6}$

(ii) $0.4\overline{7}$

(iii) $0.\overline{001}$

Solution :

(i) Let $x=0.\overline{6}\Rightarrow x=0.6666.........(a)$

We need to multiply both sides by 10 to get

$10x=6.6666.....\qquad ....(b)$

We need to subtract (a)from (b), to get

$$\begin{aligned} 10x &= 6.6666..... \\ -x &= 0.6666..... \\ \hline 9x &= 6 \end{aligned}$$

We can also write $9x=6$ as $x=\frac{6}{9}$ or $x=\frac{2}{3}$.

Therefore, on converting $0.\overline{6}$ in the $\frac{p}{q}$ form, we get the answer a $\frac{2}{3}$.

(ii) $\text{Let } x=0.4\overline{7} \Rightarrow x=0.47777.......(a)$

We need to multiply both sides by 10 to get

$10x=4.7777.....(b)$

We need to subtract (a)from (b), to get

$$\begin{aligned} 10x &= 4.7777..... \\ -x &= 0.4777.... \\ \hline 9x &= 4.3 \end{aligned}$$

We can also write $9x=4.3$ as $x=\frac{4.3}{9}$ or $x=\frac{43}{90}$.

Therefore, on converting $0.4\overline{7}$ in the $\frac{p}{q}$ form, we get the answer as $\frac{43}{90}$.

(iii) $\text{Let } x=0.\overline{001} \Rightarrow x=0.001001........(a)$

We need to multiply both sides by 1000 to get

$1000x=1.001001..... \qquad ....(b)$

We need to subtract (a)from (b), to get

$$\begin{aligned} 1000x &= 1.001001..... \\ -x &= 0.001001.... \\ \hline 999x &= 1 \end{aligned}$$

We can also write $999x = 1$ as $x = \frac{1}{999}$

Therefore, on converting $0.\overline{001}$ in the $\frac{p}{q}$ form, we get the answer as $\frac{1}{999}$ 4

Question 4. Express $0.99999....$ in the form $\frac{p}{q}$. Are you surprised by your answer? Discuss why the answer makes sense with your teacher and classmates.

Solution :

Let $x = 0.99999......(a)$

We need to multiply both sides by 10 to get

$10x = 9.9999......(b)$

We need to subtract (a)from (b), to get

$$\begin{array}{l} 10x = 9.99999..... \\ \underline{-x = 0.99999....} \\ \quad 9x = 9 \end{array}$$

We can also write $9x = 9$ as $x = \frac{9}{9}$ or $x = 1$.

Therefore, on converting $0.99999....$ in the $\frac{p}{q}$ form, we get the answer as 1 .

Yes, at a glance we are surprised at our answer.

But the answer makes sense when we observe that 0.9999......... goes on forever. SO there is not gap between 1 and 0.9999....... and hence they are equal.

Question 5. What can the maximum number of digits be in the recurring block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

Solution :

We need to find the number of digits in the recurring block of $\frac{1}{17}$.

Let us perform the long division to get the recurring block of $\frac{1}{17}$.

We need to divide 1 by 17, to get

We can observe that while dividing 1 by 17 we got the remainder as 1, which will continue to be 1 after carrying out 16 continuous divisions.

Therefore, we conclude that $\frac{1}{17} = 0.0588235294117647.....$ or $\frac{1}{17} = 0.\overline{0588235294117647}$, which is a non-terminating decimal and recurring decimal.

Question 6. Look at several examples of rational numbers in the form $\frac{p}{q}$ (q ≠ 0), where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

Solution :

Let us consider the examples of the form $\frac{p}{q}$ that are terminating decimals.

$\frac{5}{2} = 2.5$

$\frac{5}{4} = 1.25$

$\frac{2}{5} = 0.4$

$\frac{2}{10} = 0.2$

$\frac{5}{16} = 0.3125$

We can observe that the denominators of the above rational numbers have powers of 2, 5 or both.

Therefore, we can conclude that the property, which q must satisfy in $\frac{p}{q}$, so that the rational number $\frac{p}{q}$ is a terminating decimal is that q must have powers of 2, 5 or both.

Question 7. Write three numbers whose decimal expansions are non-terminating non-recurring.

Solution : The three numbers that have their expansions as non-terminating on recurring decimal are given below.

0.04004000400004 ....

0.07007000700007 ....

0.013001300013000013 ....

Question 8. Find three different irrational numbers between the rational numbers and $\frac{5}{7}$ and $\frac{9}{11}$.

Solution :

Let us convert $\frac{5}{7}$ and $\frac{9}{11}$ into decimal form, to get

$\frac{5}{7} = 0.714285....$ and $\frac{9}{11} = 0.818181....$

Three irrational numbers that lie between $0.714285....$ and $0.818181....$ are:

0.73073007300073 ....

0.74074007400074 ....

0.76076007600076 ....

Question 9. Classify the following numbers as rational or irrational :

(i)$\sqrt{23}$

(ii)$\sqrt{225}$

(iii) 0.3796

(iv) 7.478478…

(v) 1.101001000100001…

Solution :

(i) $\sqrt{23}$

We know that on finding the square root of 23, we will not get an integer.

Therefore, we conclude that $\sqrt{23}$ is an irrational number.

(ii) $\sqrt{225}$

We know that on finding the square root of 225, we get 15, which is an integer.

Therefore, we conclude that $\sqrt{225}$ is a rational number.

(iii) 0.3796

We know that 0.3796 can be converted into $\frac{p}{q}$.

While, converting 0.3796 into $\frac{p}{q}$ form, we get

$0.3796 = \frac{3796}{10000}$.

The rational number $\frac{3796}{10000}$ can be converted into lowest fractions, to get $\frac{949}{2500}$.

We can observe that 0.3796 can be converted into a rational number.

Therefore, we conclude that 0.3796 is a rational number.

(iv) 7.478478....

We know that 7.478478.... is a non-terminating recurring decimal, which can be converted into $\frac{p}{q}$ form.

While, converting 7.478478.... into $\frac{p}{q}$ form, we get

$x = 7.478478.... \quad ....(a)$

$1000x = 7478.478478.......(b)$

While, subtracting (a) from (b), we get

$$\begin{aligned} 1000x &= 7478.478478.... \\ -x &= \quad 7.478478.... \\ \hline 999x &= 7471 \end{aligned}$$

We know that $999x = 7471$ can also be written as $x = \frac{7471}{999}$.

Therefore, we conclude that 7.478478.... is a rational number.

(v)1.101001000100001 ....

We can observe that the number 1.101001000100001.... is a non-terminating on recurring decimal.

We know that non-terminating and non-recurring decimals cannot be converted into $\frac{p}{q}$ form.

Therefore, we conclude that 1.101001000100001.... is an irrational number.