CHAPTER 11 SURFACE AREAS AND VOLUMES

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1. Find the volume of a sphere whose radius is

(i) 7 cm (ii) 0.63 m

(Assume π =22/7)

Solution:

(i) Radius of the sphere, r = 7 cm

Using, Volume of the sphere = $(4/3)\ \pi r^3$

$= (4/3)\times(22/7)\times 7^3$

$= 4312/3$

Hence, volume of the sphere is $4312/3\ cm^3$

(ii) Radius of the sphere, r = 0.63 m

Using, volume of sphere = $(4/3)\ \pi r^3$

= $(4/3)\times(22/7)\times0.63^3$

= 1.0478

Hence, volume of the sphere is 1.05 m^3 (approx).

2. Find the amount of water displaced by a solid spherical ball of diameter

(i) 28 cm (ii) 0.21 m

(Assume π =22/7)

Solution:

(i) Diameter = 28 cm

Radius, r = 28/2 cm = 14cm

Volume of the solid spherical ball = $(4/3)\ \pi r^3$

Volume of the ball = $(4/3)\times(22/7)\times14^3$ = 34496/3

Hence, volume of the ball is $34496/3\ cm^3$

(ii) Diameter = 0.21 m

Radius of the ball =0.21/2 m= 0.105 m

Volume of the ball = $(4/3)\pi r^3$

Volume of the ball = $(4/3)\times (22/7)\times 0.105^3$ m^3

Hence, volume of the ball = 0.004851 m^3

3. The diameter of a metallic ball is 4.2cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm^3? (Assume π=22/7)

Solution:

Given,

Diameter of a metallic ball = 4.2 cm

Radius(r) of the metallic ball, r = 4.2/2 cm = 2.1 cm

Volume formula = $4/3\ \pi r^3$

Volume of the metallic ball = $(4/3)\times(22/7)\times 2.1$ cm^3

Volume of the metallic ball = 38.808 cm^3

Now, using the relationship between density, mass and volume,

Density = Mass/Volume

Mass = Density × volume

= (8.9×38.808) g

= 345.3912 g

Mass of the ball is 345.39 g (approx).

4. The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

Solution:

Let the diameter of the earth be "d". Therefore, the radius of the earth will be d/2.

Diameter of the moon will be d/4, and the radius of the moon will be d/8.

Find the volume of the moon.

Volume of the moon = $(4/3)\ \pi r^3 = (4/3)\ \pi\ (d/8)^3 = 4/3\pi(d^3/512)$

Find the volume of the earth

Volume of the earth = $(4/3)\ \pi r^3 = (4/3)\ \pi\ (d/2)^3 = 4/3\pi(d^3/8)$

Fraction of the volume of the earth is the volume of the moon

$$\text{Volume of the moon/ volume of the earth} = \frac{\frac{4}{3}\pi(\frac{d^3}{512})}{\frac{4}{3}\pi(\frac{d^3}{8})} = 8/512 = 1/64$$

Answer: Volume of the moon is of the 1/64 volume of the earth.

5. How many litres of milk can a hemispherical bowl of diameter 10.5cm hold? (Assume π = 22/7)

Solution:

Diameter of the hemispherical bowl = 10.5 cm

Radius of the hemispherical bowl, r = 10.5/2 cm = 5.25 cm

Formula for volume of the hemispherical bowl = $(2/3)\ \pi r^3$

Volume of the hemispherical bowl = $(2/3)\times(22/7)\times5.25^3$ = 303.1875

Volume of the hemispherical bowl is 303.1875 cm^3

Capacity of the bowl = (303.1875)/1000 L = 0.303 litres(approx.)

Therefore, the hemispherical bowl can hold 0.303 litres of milk.

6. A hemispherical tank is made up of an iron sheet 1cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank. (Assume π = 22/7)

Solution:

Inner Radius of the tank, (r) = 1m

Outer Radius (R) = 1.01m

Volume of the iron used in the tank = $(2/3)\ \pi(R^3 - r^3)$

Put values,

Volume of the iron used in the hemispherical tank = $(2/3)\times(22/7)\times(1.01^3 - 1^3)$ = 0.06348

So, volume of the iron used in the hemispherical tank is 0.06348 m^3.

7. Find the volume of a sphere whose surface area is 154 cm^2. (Assume π = 22/7)

Solution:

Let r be the radius of a sphere.

Surface area of the sphere = $4\pi r^2$

$4\pi r^2 = 154\ cm^2$ (given)

$r^2 = (154\times7)/(4\times22)$

r = 7/2

The radius is 7/2 cm.

Now,

Volume of the sphere = $(4/3)\ \pi r^3$

$$\text{Volume of the sphere} = (4/3)\times(22/7)\times(7/2)^3 = 179\frac{2}{3}$$

$$\text{Volume of the sphere is } 179\frac{2}{3}\ cm^3$$

8. A dome of a building is in the form of a hemisphere. From inside, it was whitewashed at the cost of Rs. 4989.60. If the cost of white-washing is 20 per square metre, find the

(i) inside surface area of the dome (ii) volume of the air inside the dome

(Assume π = 22/7)

Solution:

(i) Cost of whitewashing the dome from inside = Rs 4989.60

Cost of whitewashing $1m^2$ area = Rs 20

CSA of the inner side of dome = 498.96/2 m^2 = 249.48 m^2

(ii) Let the inner radius of the hemispherical dome be r.

CSA of the inner side of dome = 249.48 m^2 (from (i))

Formula to find CSA of a hemisphere = $2\pi r^2$

$2\pi r^2$ = 249.48

$2\times(22/7)\times r^2$ = 249.48

r^2 = (249.48×7)/(2×22)

r^2 = 39.69

r = 6.3

So, the radius is 6.3 m.

Volume of air inside the dome = Volume of hemispherical dome

Using the formula, the volume of the hemisphere = $2/3\ \pi r^3$

= (2/3)×(22/7)×6.3×6.3×6.3

= 523.908

= 523.9(approx.)

Answer: The volume of air inside the dome is 523.9 m^3.

9. Twenty-seven solid iron spheres, each of radius r and surface area S are melted to form a sphere with surface area S'. Find the

(i) radius r′ of the new sphere,

(ii) ratio of Sand S′.

Solution:

Volume of the solid sphere = $(4/3)\pi r^3$

Volume of twenty seven solid sphere = $27\times(4/3)\pi r^3 = 36\ \pi\ r^3$

(i) New solid iron sphere radius = r′

Volume of this new sphere = $(4/3)\pi(r')^3$

$(4/3)\pi(r')^3 = 36\ \pi\ r^3$

$(r')^3 = 27r^3$

$r' = 3r$

Radius of the new sphere will be 3r (thrice the radius of the original sphere)

(ii) Surface area of the iron sphere of radius r, $S = 4\pi r^2$

Surface area of the iron sphere of radius $r' = 4\pi\ (r')^2$

Now

$S/S' = (4\pi r^2)/(\ 4\pi\ (r')^2)$

$S/S' = r^2/(3r')^2 = 1/9$

The ratio of S and S′ is 1: 9.

10. A capsule of medicine is in the shape of a sphere of diameter 3.5mm. How much medicine (in mm^3) is needed to fill this capsule? (Assume $\pi = 22/7$)

Solution:

Diameter of the capsule = 3.5 mm

Radius of the capsule, say r = diameter/ 2 = (3.5/2) mm = 1.75mm

Volume of the spherical capsule = $4/3\ \pi r^3$

Volume of the spherical capsule = $(4/3)\times(22/7)\times(1.75)^3 = 22.458$

Answer: The volume of the spherical capsule is 22.46 mm^3.