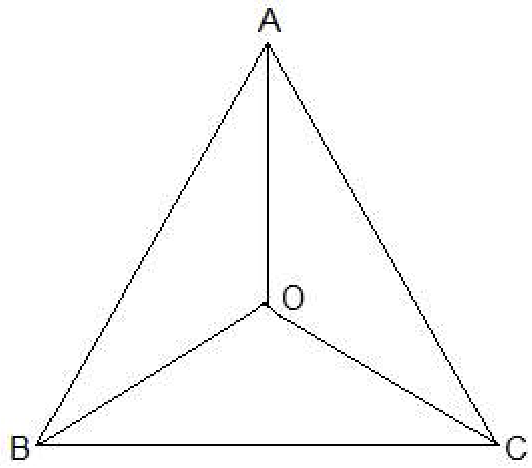
CHAPTER 7 TRIANGLES

EXERCISE 7.2 PAGE:97

1. In an isosceles triangle ABC, with AB = AC, the bisectors of ∠B and ∠C intersect each other at O. Join A to O. Show that:

(i) OB = OC (ii) AO bisects ∠A



Solution:

Given:

AB = AC and

the bisectors of ∠B and ∠C intersect each other at O

(i) Since ABC is an isosceles with AB = AC,

∠B = ∠C

½ ∠B = ½ ∠C

⇒ ∠OBC = ∠OCB (Angle bisectors)

∴ OB = OC (Side opposite to the equal angles are equal.)

(ii) In ΔAOB and ΔAOC,

AB = AC (Given in the question)

AO = AO (Common arm)

OB = OC (As Proved Already)

So, ΔAOB ≅ ΔAOC by SSS congruence condition.

BAO = CAO (by CPCT)

Thus, AO bisects ∠A.

2. In ΔABC, AD is the perpendicular bisector of BC (see Fig. 7.30). Show that ΔABC is an isosceles triangle in which AB = AC.

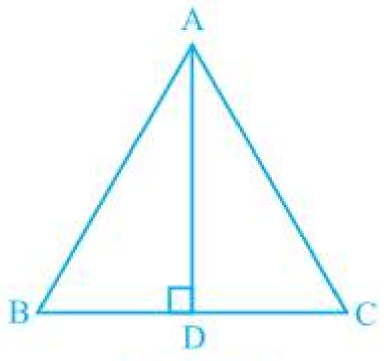


Fig. 7.30

Solution:

It is given that AD is the perpendicular bisector of BC

To prove:

AB = AC

Proof:

In ΔADB and ΔADC,

AD = AD (It is the Common arm)

∠ADB = ∠ADC

BD = CD (Since AD is the perpendicular bisector)

So, ΔADB ≅ ΔADC by **SAS congruency criterion**.

Thus,

AB = AC (by CPCT)

3. ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (see Fig. 7.31). Show that these altitudes are equal.

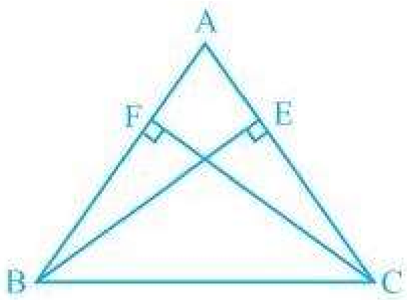


Fig. 7.31

Solution:

Given:

(i) BE and CF are altitudes.

(ii) AC = AB

To prove:

BE = CF

Proof:

Triangles ΔAEB and ΔAFC are similar by AAS congruency since

∠A = ∠A (It is the common arm)

∠AEB = ∠AFC (They are right angles)

AB = AC (Given in the question)

∴ ΔAEB ≅ ΔAFC and so, BE = CF (by CPCT).

4. ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (see Fig. 7.32). Show that

(i) ΔABE ≅ ΔACF

(ii) AB = AC, i.e., ABC is an isosceles triangle.

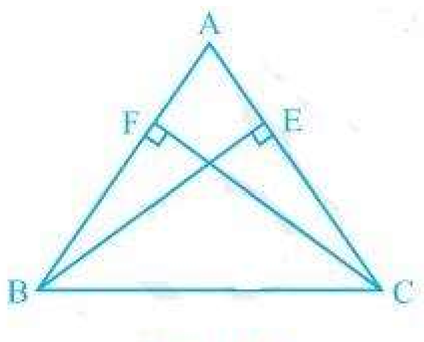


Fig. 7.32

Solution:

It is given that BE = CF

(i) In ΔABE and ΔACF,

∠A = ∠A (It is the common angle)

∠AEB = ∠AFC (They are right angles)

BE = CF (Given in the question)

∴ ΔABE ≅ ΔACF by **AAS congruency condition**.

(ii) AB = AC by CPCT and so, ABC is an isosceles triangle.

5. ABC and DBC are two isosceles triangles on the same base BC (see Fig. 7.33). Show that ∠ABD = ∠ACD.

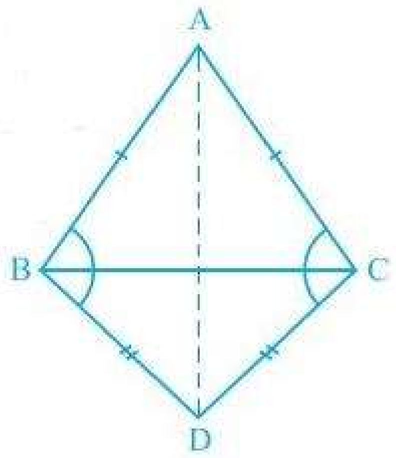


Fig. 7.33

Solution:

In the question, it is given that ABC and DBC are two isosceles triangles.

We will have to show that ∠ABD = ∠ACD

Proof:

Triangles ΔABD and ΔACD are similar by SSS congruency since

AD = AD (It is the common arm)

AB = AC (Since ABC is an isosceles triangle)

BD = CD (Since BCD is an isosceles triangle)

So, ΔABD ≅ ΔACD.

∴ ∠ABD = ∠ACD by CPCT.

6. ΔABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB (see Fig. 7.34). Show that ∠BCD is a right angle.

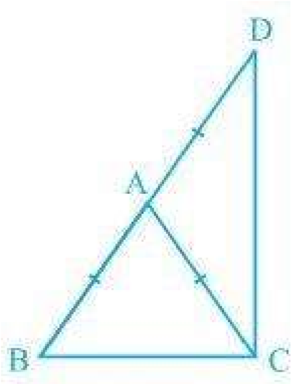


Fig. 7.34

Solution:

It is given that AB = AC and AD = AB

We will have to now prove ∠BCD is a right angle.

Proof:

Consider ΔABC,

AB = AC (It is given in the question)

Also, ∠ACB = ∠ABC (They are angles opposite to the equal sides and so, they are equal)

Now, consider ΔACD,

AD = AB

Also, ∠ADC = ∠ACD (They are angles opposite to the equal sides and so, they are equal)

Now,

In ΔABC,

$\angle CAB + \angle ACB + \angle ABC = 180°$

So, $\angle CAB + 2\angle ACB = 180°$

$\Rightarrow \angle CAB = 180° – 2\angle ACB$ — (i)

Similarly, in ΔADC,

$\angle CAD = 180° – 2\angle ACD$ — (ii)

also,

$\angle CAB + \angle CAD = 180°$ (BD is a straight line.)

Adding (i) and (ii) we get,

$\angle CAB + \angle CAD = 180° – 2\angle ACB+180° – 2\angle ACD$

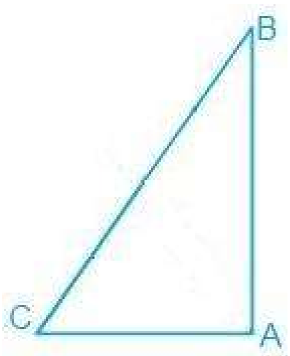
$\Rightarrow 180° = 360° – 2\angle ACB-2\angle ACD$

⇒ 2(∠ACB+∠ACD) = 180°

⇒ ∠BCD = 90°

7. ABC is a right-angled triangle in which ∠A = 90° and AB = AC. Find ∠B and ∠C.

Solution:



In the question, it is given that

∠A = 90° and AB = AC

AB = AC

⇒ ∠B = ∠C (They are angles opposite to the equal sides and so, they are equal)

Now,

∠A+∠B+∠C = 180° (Since the sum of the interior angles of the triangle)

∴ 90° + 2∠B = 180°

⇒ 2∠B = 90°

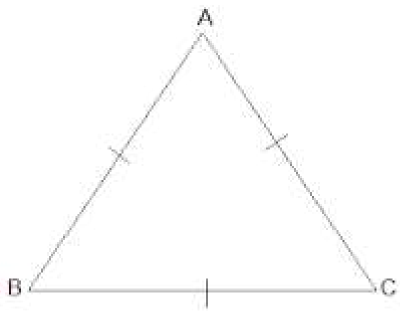
⇒ ∠B = 45°

So, ∠B = ∠C = 45°

8. Show that the angles of an equilateral triangle are 60° each.

Solution:

Let ABC be an equilateral triangle as shown below:



Here, BC = AC = AB (Since the length of all sides is same)

⇒ ∠A = ∠B =∠C (Sides opposite to the equal angles are equal.)

Also, we know that

∠A+∠B+∠C = 180°

⇒ 3∠A = 180°

⇒ ∠A = 60°

∴ ∠A = ∠B = ∠C = 60°

So, the angles of an equilateral triangle are always 60° each.