Exercise 2.1, Page: 26

Question1. Find the principal value of $\sin^{-1}$ (-1/2)

Solution

Let $\sin^{-1}\left(-\frac{1}{2}\right) = y$ Then $\sin y = -\frac{1}{2} = -\sin\left(\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right)$

We know that the range of the principal value branch of $\sin^{-1}$ is

$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\sin\left(-\frac{\pi}{2}\right) = -\frac{1}{2}$

Therefore, the principal value of $\sin^{-1}\left(-\frac{1}{2}\right) is -\frac{\pi}{6}$

Question2. $\cos^{-1}$(√3/2)

Solution

Let $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = y,$ Then $\cos y = \frac{\sqrt{3}}{2} = \cos\left(\frac{\pi}{6}\right)$

We know that the range of the principal value branch of $\cos^{-1}$ is

$[0, \pi]$ and $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

Therefore the priciple value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) is \frac{\pi}{6}$

Question3. cosec^{-1}(2)

Solution

Let cosec^{-1} (2) = y. Then, $\cos ecy = 2 = \cos ec\left(\frac{\pi}{6}\right)$

We know that the range of the principal value branch of cosec^{-1} is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

Therefore, the principal value of $\cos ec^{-1}(2)$ is $\frac{\pi}{6}$

Question4. $\tan^{-1}$(-√3)

Solution

Let $\tan^{-1}\left(-\sqrt{3}\right) = y$. Then $\tan y = -\sqrt{3} = -\frac{\tan\pi}{3} = \tan\left(-\frac{\pi}{3}\right)$

We know that the range of the principal value branch of $\tan^{-1}$ is

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\tan\left(-\frac{\pi}{3}\right) is - \sqrt{3}$

Therefore, the principal value of $\tan^{-1}\left(\sqrt{3}\right)$ is $-\frac{\pi}{3}$

Question5. cos⁻¹(-1/2)

Solution :

Let $\cos^{-1}\left(-\frac{1}{2}\right) = y$ Then $\cos y = -\frac{1}{2} = -\cos\left(\frac{\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right)$

We know that the range of the principal value branch of $\cos^{-1}$ is [0,pi] and $\cos\left(2\frac{\pi}{3}\right) = \frac{1}{2}$

Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is $\frac{2\pi}{3}$

Question6. tan⁻¹ (-1)

Solution :

Let $\tan^{-1}(-1) = y$. Then tan y = -1 = $-\tan\left(\frac{\pi}{4}\right) = \tan\left(-\frac{\pi}{4}\right)$

We know that the range of the principal value branch of $\tan^{-1}$ is

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\tan\left(-\frac{\pi}{4}\right) = -1$

Therefore, the principal value of $\tan^{-1}(-1)$ is $\frac{\pi}{4}$

Question7. sec⁻¹(2√3)

Let $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = y$, Then $\sec y = \frac{2}{\sqrt{3}} = \sec\left(\frac{\pi}{6}\right)$

We know that the range of the principal value branch of $\sec^{-1}$ is

$[0, \pi] - \left\{\frac{\pi}{2}\right\}$ and $\sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}$

Therefore, the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) is \frac{\pi}{6}$

Solution :

Question8. $\cot^{-1}(\sqrt{3})$

Solution

Let $\cot^{-1}(\sqrt{3}) = y$ Then $\cot y = \sqrt{3} \quad = \cot\left(\frac{\pi}{6}\right)$

We know that the range of the principal value branch of $\cot^{-1}$ is $(0,\pi)$ and

$\cot\left(\frac{\pi}{6}\right) = \sqrt{3}$

Therefore, the principal value of $\cot^{-1}(\sqrt{3})$ is $\frac{\pi}{6}$

Question9. $\cos^{-1}(-1/\sqrt{2})$

Solution

Let $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = y$ Then $\cos y = -\frac{1}{\sqrt{2}} = -\cos\left(\frac{\pi}{4}\right) = \cos\left(\pi - \frac{\pi}{4}\right) = \cos\left(\frac{3\pi}{4}\right)$

We know that the range of the principal value branch of $\cos^{-1}$ is $[0,\pi]$ and

$\cos\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is $\frac{3\pi}{4}$

Question10. $\text{cosec}^{-1}(-\sqrt{2})$

Solution :

Let $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)=y$. Then, $\operatorname{cosec} y=-\sqrt{2}=-\operatorname{cosec}\left(\frac{\pi}{4}\right)=\operatorname{cosec}\left(-\frac{\pi}{4}\right)$

We know that the range of the principal value branch of $\operatorname{cosec}^{-1}$ is

$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$ and $\operatorname{cosec}\left(-\frac{\pi}{4}\right)=-\sqrt{2}$

Therefore, the principal value of $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$ is $-\frac{\pi}{4}$

Find the value of the following:

Question11. $\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$

Solution

Let $\tan^{-1}(1)$ = x Then tan x= 1 = tan $\frac{\pi}{4}$

$\therefore \tan^{-1}(1)=\frac{\pi}{4}$

Let $\cos^{-1}\left(-\frac{1}{2}\right)=y$ Then, $\cos y=-\frac{1}{2}=-\cos\left(\frac{\pi}{3}\right)=\cos\left(\pi-\frac{\pi}{3}\right)=\cos\left(\frac{2\pi}{3}\right)$

$\therefore \cos^{-1}\left(-\frac{1}{2}\right)=\frac{2\pi}{3}$

Let $\sin^{-1}\left(-\frac{1}{2}\right)=z$. Then $\sin z=-\frac{1}{2}=-\sin\left(\frac{\pi}{6}\right)=\sin\left(-\frac{\pi}{6}\right)$

$\therefore \sin^{-1}\left(-\frac{1}{2}\right)=-\frac{\pi}{6}$

$\therefore \tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$

$=\frac{\pi}{4}+\frac{2\pi}{3}-\frac{\pi}{6}$

$=\frac{3\pi+8\pi-2\pi}{12}=\frac{9\pi}{12}=\frac{3\pi}{4}$

Question12. $\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$

Solution

Let cos^(-1) (1/2) = x. Then, $\cos x = \frac{1}{2} = \cos(\pi/3)$

$\therefore \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

Let $\sin^{-1}\left(\frac{1}{2}\right) = y$, Then $\sin y = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$

$\therefore \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\therefore \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + \frac{2\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$

Question13.

Find the value of if $\sin^{-1} x = y$, then

A) $0 \leq y < \pi$

B) $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

c) $0 < y < \pi$

d) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

Solution

It is given that $\sin^{-1} x = y$.

We know that the range of the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Therefore, $\left[-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}\right]$

Therefore, option (B) is correct.

Question14.

Find the value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ is equal to

(A) π

(B) $-\frac{\pi}{3}$

(C) $\frac{\pi}{3}$

(D) $\frac{2\pi}{3}$

Solution

Let $\tan^{-1} = x$, Then $\tan x = \sqrt{3} = \dfrac{\tan \pi}{3}$

We know that the range of the principle value branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\therefore \tan^{-1}\sqrt{3} = \dfrac{\pi}{3}$

Let $\sec^{-1}(-2) = y$ Then , $\sec y = (-2) = -\sec\left(\frac{\pi}{3}\right) = \sec\left(\pi - \frac{\pi}{3}\right) = \sec \dfrac{2\pi}{3}$

We know that the range of the principle value branch of $\sec^{-1}$ is $[0, \pi]$ - $\left\{\frac{\pi}{2}\right\}$

$\therefore \sec^{-1}(-2) = \dfrac{2\pi}{3}$

Hence, $\tan^{-1}\left(\sqrt{3}\right) - \sec^{-1}(-2) = \dfrac{\pi}{3} - \dfrac{2\pi}{3} = -\dfrac{\pi}{3}$

Therefore, option (B) is correct.

Exercise 2.2 Page: 29

Question1. $3\sin^{-1}x = \sin^{-1}(3x - 4x^3), x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Solution :

To prove : $3\sin^{-1}x = \sin^{-1}(3x - 4x^3), x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Let x = sinθ. Then, $\sin^{-1}x = 0$

We have

R.H.S = $\sin^{-1}(3x - 4x^3) = \sin^{-1}(3\sin\theta - 4\sin^3\theta)$

$= \sin^{-1}(\sin 3\theta)$`

= 3θ

$= 3\sin^{-1}x$

L.H.S

Proved.

Question2. $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x), x \in \left[\frac{1}{2}, 1\right]$

Solution :]

Let x = cosθ. Then, cos^{-1} x =θ.

We have,

R.H.S = $\cos^{-1}(4x^3 - 3x)$

$= \cos^{-1}(4\cos^3\theta - 3\cos\theta)$

$= \cos^{-1}(\cos 3\theta)$

$= 3\theta$

$= 3\cos^{-1}x$

\\\ L.H.S

Proved.

Question3. $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

Solution

L.H.S =$\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24}$

$= \frac{\tan^{-1}\left(\frac{2}{11} + \frac{7}{24}\right)}{1 - \frac{2}{11} \cdot \frac{7}{24}} \quad \left[\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}\right]$

= tan^(-1) $\frac{\frac{48+77}{11\times24}}{\frac{11\times24-14}{11\times24}}$

$= \tan^{-1}\frac{48+77}{264-14} = \tan^{-1}\frac{125}{250} = \tan^{-1}\frac{1}{2} =$ R.H.S

Proved.

Question4. $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

Solution :

L.H.S = $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}$

$=\tan^{-1}\frac{2.\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2}+\tan^{-1}\frac{1}{7}$ $\left[2\tan^{-1}x=\tan^{-1}\frac{2x}{1-x^2}\right]$

$=\tan^{-1}\frac{1}{\left(\frac{3}{4}\right)}+\tan^{-1}\frac{1}{7}$

$=\tan^{-1}\frac{4}{3}+\tan^{-1}\frac{1}{7}$

$=\tan^{-1}\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}.\frac{1}{7}}$ $\left[\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}\right]$

$=\tan^{-1}\frac{\frac{28+3}{21}}{\frac{21-4}{21}}$

$=\tan^{-1}\frac{31}{17}$ = R.H.S

Write the following functions in the simplest form:

Question3. : $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}, x\neq 0$

Solution :

$\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$

Put $x=\tan\theta\Rightarrow\theta=\tan^{-1}x$

$\therefore\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}=\tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$

$=\tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)=\tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$

$=\tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$

$=\tan^{-1}\left(\tan\frac{\theta}{2}\right)=\frac{\theta}{2}=\frac{1}{2}\tan^{-1}x$

Question4. $\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right), x < \pi$

Solution

$$\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right), x < \pi$$

$$\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right) = \tan^{-1}\left(\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}\right)$$

$$= \tan^{-1}\left(\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\right) = \tan^{-1}\left(\tan\frac{x}{2}\right)$$

$$= \frac{x}{2}$$

Question5 $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right), 0 < x < \pi$

Solution

$$\tan^{-1}\frac{\cos x-\sin x}{\cos x+\sin x}$$

Dividing cos x inside

$$=\tan^{-1}\left[\frac{\frac{\cos x-\sin x}{\cos x}}{\frac{\cos x+\sin x}{\cos x}}\right]$$

$$=\tan^{-1}\left[\frac{\frac{\cos x}{\cos x}-\frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x}+\frac{\sin x}{\cos x}}\right]$$

$$=\tan^{-1}\frac{1-\tan x}{1+\tan x}$$

$$=\tan^{-1}\left[\frac{1-\tan x}{1+1.\tan x}\right]$$

$$=\tan^{-1}\left[\frac{\frac{\tan\pi}{4}-\tan x}{1+\frac{\tan\pi}{4}.\tan x}\right]$$ (As $\tan\frac{\pi}{4}$ = 1)

$$=\tan^{-1}\tan\left(\frac{\pi}{4}-x\right)$$

$$=\frac{\pi}{4}-x$$

Question6. $\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}$, |x| < a

Solution :

$$\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}$$

Put $x=a\sin\,\theta\Rightarrow\frac{x}{a}=\sin\theta\Rightarrow\theta=\sin^{-1}\left(\frac{x}{a}\right)$

$$\therefore\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}=\tan^{-1}\left(\frac{a\sin\theta}{\sqrt{a^2-a^2\sin^2\theta}}\right)$$

$$=\tan^{-1}\left(\frac{a\sin\theta}{a\sqrt{1-\sin^2\theta}}\right)=\tan^{-1}\left(\frac{a\sin\theta}{a\cos\theta}\right)$$

$$=\tan^{-1}(\tan\theta)=\theta=\sin^{-1}\frac{x}{a}$$

Question7. $\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right), a > 0; \frac{-a}{\sqrt{3}} \leq x \frac{a}{\sqrt{3}}$

Solution

$$\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right)$$

Put $x = a\tan\theta \Rightarrow \frac{x}{a} = \tan\theta \Rightarrow \theta = \tan^{-1}\frac{x}{a}$

$$\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right) = \tan^{-1}\left(\frac{3a^2.\,a\tan\theta - a^3\tan^3\theta}{a^3 - 3a.\,a^2\tan^2\theta}\right)$$

$$\tan^{-1}\left(\frac{3a^3\tan\theta - a^3\tan^3\theta}{a^3 - 3a^3\tan^2\theta}\right)$$

$$\tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right)$$

$$= \tan^{-1}(\tan 3\theta)$$

$$= 3\theta$$

$$= 3\tan^{-1}\frac{x}{a}$$

Find the values of each of the following:

Question8. $\tan^{-1}\left[2\cos\left(2\frac{\sin^{-1}1}{2}\right)\right]$

Solution

Let $\sin^{-1}\frac{1}{2} = x$. Then $\sin x = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$

$$\therefore \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$$

$$\therefore \tan^{-1}\left[2\cos\left(2\frac{\sin^{-1}1}{2}\right)\right] = \tan^{-1}\left[2\cos\left(2 \times \frac{\pi}{6}\right)\right]$$

$$= \tan^{-1}\left[2\cos\frac{\pi}{3}\right] = \tan^{-1}\left[2 \times \frac{1}{2}\right]$$

$$= \tan^{-1}1 = \frac{\pi}{4}$$

Question9. $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right], |x|<1, y>0 \text{ and } xy<1$

Solution :

Let $x = \tan\theta$. Then, $\theta = \tan^{-1} x$.

$$\therefore \sin^{-1}\frac{2x}{1+x^2}=\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)=\sin^{-1}(\sin 2\theta)=2\theta=2\tan^{-1}x$$

Let $y = \tan\Phi$. Then, $\Phi = \tan^{-1} y$.

$$\therefore \cos^{-1}\frac{1-y^2}{1+y^2}=\cos^{-1}\left(\frac{1-\tan^2\phi}{1+\tan^2\phi}\right)=\cos^{-1}(\cos 2\phi)=2\phi=2\tan^{-1}y$$

$$\therefore \tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right]$$

$$=\tan\frac{1}{2}\left[2\tan^{-1}x+2\tan^{-1}y\right]$$

$$=\tan\left[\tan^{-1}x+\tan^{-1}y\right]$$

$$=\tan\left[\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$=\frac{x+y}{1-xy}$$

Find the values of each of the expressions in Exercises 16 to 18.

Question10. $\sin^{-1}\left(\sin 2\frac{\pi}{3}\right)$

Solution

$\sin^{-1}\left(\sin\ 2\frac{\pi}{3}\right)$

We know that $\sin^{-1}(\sin x) = x$ if x in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, which is the principal value branch of $\sin^{-1}x$.

Here, $2\frac{\pi}{3} \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

Now $\sin^{-1}\left(\sin 2\frac{\pi}{3}\right)$ can be written as

$\sin^{-1}\left(\sin\ \frac{2\pi}{3}\right) = \sin^{-1}\left[\sin\left(\pi - \frac{2\pi}{3}\right)\right] = \sin^{-1}\left(\sin\ \frac{\pi}{3}\right)$ where $\frac{\pi}{3} \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$\therefore \sin^{-1}\left(\sin\ \frac{2\pi}{2}\right) = \sin^{-1}\left(\sin\ \frac{\pi}{3}\right) = \frac{\pi}{3}$

Question11. $\tan^{-1}\left(\tan\ \frac{3\pi}{4}\right)$

Solution

$\tan^{-1}\left(\tan\ \frac{3\pi}{4}\right)$

We know that $\tan^{-1}(\tan x) = x$ if x in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ which is the principal value branch of $\tan^{-1}x$.

Here $\frac{3\pi}{4} \notin \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

Now , $\tan^{-1}\left(\tan\ \frac{3\pi}{4}\right)$ can be witten as

$\tan^{-1}\left(\tan\ \frac{3\pi}{4}\right) = \tan^{-1}\left[-\tan\ \left(\frac{-3\pi}{4}\right)\right] = \tan^{-1}\left[-\tan\left(\pi - \frac{\pi}{4}\right)\right]$

$= \tan^{-1}\left[-\tan\ \frac{\pi}{4}\right] = \tan^{-1}\left[\tan\left(-\frac{\pi}{4}\right)\right]$ where $-\frac{\pi}{4} \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$\therefore \tan^{-1}\left(\tan\ \frac{3\pi}{4}\right) = \tan^{-1}\left[\tan\left(\frac{-\pi}{4}\right)\right] = \frac{-\pi}{4}$

Question12. $\tan\left(\sin^{-1}\ \frac{3}{5} + \cot^{-1}\ \frac{3}{2}\right)$

Solution :

Let $\sin^{-1}\frac{3}{5}=x$. Then $\sin x=\frac{3}{5}\Rightarrow\cos x=\sqrt{1-\sin^2 x}=\frac{4}{5}\Rightarrow\sec x=\frac{5}{4}$

$\therefore\tan x=\sqrt{\sec^2 x-1}=\sqrt{\frac{25}{16}-1}=\frac{3}{4}$

$\therefore x=\frac{\tan^{-1}3}{4}$

$\therefore\sin^{-1}\frac{3}{5}=\frac{\tan^{-1}3}{4}$ ...(i)

Now $\cot^{-1}\frac{3}{2}=\tan^{-1}\frac{2}{3}$ ...(ii) $\left[\tan^{-1}\frac{1}{x}=\cot^{-1}x\right]$

Hence, $\tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$

$=\tan\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{2}{3}\right)$ [Using i and ii]

$=\tan\left(\tan^{-1}\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\cdot\frac{2}{3}}\right)$ $\left[\tan^{-1}x+\tan^{-1}y=\frac{\tan^{-1}(x+y)}{1}-xy)\right]$

$=\tan\left(\tan^{-1}\frac{9+8}{12-6}\right)$

$=\tan\left(\tan^{-1}\frac{17}{6}\right)=\frac{17}{6}$

Question13.

Find the values of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ is equal to

(A) $\frac{7\pi}{6}$

(B) $\frac{5\pi}{6}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{6}$

Solution

We know that $\cos^{-1}(\cos x) = x$ if x in $[0, \pi]$, which is the principal value branch of $\cos^{-1}x$.

Here $\frac{7\pi}{6} \notin x \in [0, \pi]$

Now $\cos^{-1}\left(\cos \frac{7\pi}{6}\right)$ can be written as

cos-1cos7π6 = cos-1cosπ+π6cos-1cos7π6 = cos-1- cosπ6 as, cosπ+θ = - cos θcos-1cos7π6 = cos-1- cosπ-5π6cos-1cos7π6 = cos-1-- cos 5π6 as, cosπ-θ = - cos θ

$\therefore \cos^{-1}\left(\cos \frac{7\pi}{6}\right) = \cos^{-1}\left(\cos \frac{5\pi}{6}\right) = \frac{5\pi}{6}$

The correct answer is B.

Question14. $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$ **is equal to:**

(A) 1/2

(B) 1/3

(C) 1/4

(D) 1

Solution

Let $\sin^{-1}\left(-\frac{1}{2}\right) = x$. Then $\sin x = \frac{-1}{2} = -\sin\frac{\pi}{6} = \sin\left(\frac{-\pi}{6}\right)$

We know that the range of the principal value branch of $\sin^{-1} is \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$\therefore \sin^{-1}\left(-\frac{1}{2}\right) = \frac{-\pi}{6}$

$\therefore \sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right) = \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right) = \sin\left(\frac{3\pi}{6}\right) = \sin\left(\frac{\pi}{2}\right) = 1$

Therefore, option (D) is correct.

Question15. $\tan^{-1}\sqrt{3} - \cot^{-1}\left(-\sqrt{3}\right)$ **is equal to:**

(A) π

(B) -π/2

(C) 0

(D) 2√3

Solution

Let $\tan^{-1}\sqrt{3} = x$. Then, $\tan x = \sqrt{3} = \tan\frac{\pi}{3}$ where $\frac{\pi}{3} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

We know that the range of the principal value branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

$\therefore \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

Let $\cot^{-1}\left(-\sqrt{3}\right) = y$.

Then, $\cot y = -\sqrt{3} = -\cot\left(\frac{\pi}{6}\right) = \cot\left(\pi - \frac{\pi}{6}\right) = \cot\frac{5\pi}{6}$ where $\frac{5\pi}{6} \in (0, \pi)$.

The range of the principal value branch of $\cot^{-1}$ is $(0, \pi)$.

$\therefore \cot^{-1}\left(-\sqrt{3}\right) = \frac{5\pi}{6}$

$\therefore \tan^{-1}\sqrt{3} - \cot^{-1}\left(-\sqrt{3}\right) = \frac{\pi}{3} - \frac{5\pi}{6} = \frac{2\pi - 5\pi}{6} = \frac{-3\pi}{6} = -\frac{\pi}{2}$

Therefore, option (B) is correct.

Exercise 2.3, Page: 31

Find the value of the following of NCERT Solutions for Class 12 Maths Chapter 2 Miscellaneous Exercise (Inverse Trigonometric Function)

Question1. $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

Solution

We know that $\cos^{-1}(\cos x) = x$ if $x \in [0, \pi]$, which is the principal value branch of $\cos^{-1}x$.

Here, $\frac{13\pi}{6} \notin [0, \pi]$

Now $\cos^{-1}\left(\cos \frac{13\pi}{6}\right)$ can be written as

$$\cos^{-1}\left(\cos \frac{13\pi}{6}\right) = \cos^{-1}\left[\cos\left(2\pi + \frac{\pi}{6}\right)\right] = \cos^{-1}\left[\cos\left(\frac{\pi}{6}\right)\right], \text{ where } \frac{\pi}{6} \in [0, \pi]$$

$$\therefore \cos^{-1}\left(\cos \frac{13\pi}{6}\right) = \cos^{-1}\left[\cos\left(\frac{\pi}{6}\right)\right] = \frac{\pi}{6}$$

Question2. $\tan^{-1}\left(\tan \frac{7x}{6}\right)$

Solution

We know that $\tan^{-1}(\tan x) = x$ if $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, which is the principal value branch of $\tan^{-1}x$.

Here $\frac{7\pi}{6} \notin \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Now $\tan^{-1}\left(\tan \frac{7\pi}{6}\right)$ can be written as

$$\tan^{-1}\left(\tan \frac{7\pi}{6}\right) = \tan^{-1}\left[\tan\left(2\pi - \frac{5\pi}{6}\right)\right] \quad [\tan(2\pi - x) = -\tan x]$$

$$= \tan^{-1}\left[-\tan\left(\frac{5\pi}{6}\right)\right] = \tan^{-1}\left[\tan\left(\frac{-5\pi}{6}\right)\right] = \tan^{-1}\left[\tan\left(\pi - \frac{5\pi}{6}\right)\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{6}\right)\right], \text{ where } \frac{\pi}{6} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\therefore \tan^{-1}\left(\tan \frac{7\pi}{6}\right) = \tan^{-1}\left(\tan \frac{\pi}{6}\right) = \frac{\pi}{6}$$

Question3. Prove that: $2\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{24}{7}$

Let $\sin^{-1}\frac{3}{5}=x$, Then $\sin x=\frac{3}{5}$

=> $\cos x=\sqrt{1-\left(\frac{3}{5}\right)^2}=\frac{4}{5}$

$\therefore \tan x=\frac{3}{4}$

$\therefore x= \tan^{-1}\frac{3}{4} \Rightarrow \sin^{-1}\frac{3}{5}=\tan^{-1}\frac{3}{4}$

Now, we have:

L.H.S = $2\sin^{-1}\frac{3}{5}=2\tan^{-1}\frac{3}{4}$

$=\tan^{-1}\left(\frac{2\times\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2}\right) \quad \left[2\tan^{-1}x=\tan^{-1}\frac{2x}{1-x^2}\right]$

$=\tan^{-1}\left(\frac{\frac{3}{2}}{\frac{16-9}{16}}\right)= \tan^{-1}\left(\frac{3}{2}\times\frac{16}{7}\right)$

$=\tan^{-1}\frac{24}{7}$ = R.H.S

Solution :

Question4. Prove that: $\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{77}{36}$

Solution

Let sin^(-1) 8/17 = x. Then $\sin x=\frac{8}{17}\Rightarrow\cos x=\sqrt{1-\left(\frac{8}{17}\right)^2}=\sqrt{\frac{225}{289}}=\frac{15}{17}$

$\therefore\tan x=\frac{8}{15}\Rightarrow x=\tan^{-1}\frac{8}{15}$

$\therefore\sin^{-1}\frac{8}{17}=\tan^{-1}\frac{8}{15}$ ....(1)

Now let $\sin^{-1}\frac{3}{5}=y$. Then $\sin y=\frac{3}{5}\Rightarrow\cos y=\sqrt{1-\left(\frac{3}{5}\right)^2}=\sqrt{\frac{16}{25}}=\frac{4}{5}$

$\therefore\tan y=\frac{3}{4}\Rightarrow\quad y=\tan^{-1}\frac{3}{4}$

$\therefore\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{3}{4}$ 2

Now, we have:

L.H.S = $\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}$

$=\tan^{-1}\frac{8}{15}+\tan^{-1}\frac{3}{4}$ [Using 1 and 2]

$=\tan^{-1}\frac{\frac{8}{15}+\frac{3}{4}}{1-\frac{8}{15}\times\frac{3}{4}}$

$=\tan^{-1}\left(\frac{32+45}{60-24}\right)$ $\left[\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}\right]$

$=\tan^{-1}\frac{77}{36}$ = R.H.S

Question5. Prove that: $\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}=\cos^{-1}\frac{33}{65}$

Solution

Let $\cos^{-1}\frac{4}{5}=x$. Then, $\cos x=\frac{4}{5}\Rightarrow\sin x=\sqrt{1-\left(\frac{4}{5}\right)^2}=\frac{3}{5}$

$\therefore\tan x=\frac{3}{4}\Rightarrow x=\ \tan^{-1}\frac{3}{4}$

$\therefore\cos^{-1}\frac{4}{5}-\ \tan^{-1}\frac{3}{4}$ ...(1)

Now let cos^(-1) $\frac{12}{13}=y$ Then $\cos y=\frac{12}{13}\Rightarrow\sin y=\frac{5}{13}$

$\therefore\tan y=\frac{5}{12}\Rightarrow y=\tan^{-1}\frac{5}{12}$

$\therefore\cos^{-1}\frac{12}{13}=\tan^{-1}\frac{5}{12}\ ---2$

Let $\cos^{-1}\frac{33}{65}=z$. Then $\cos z=\frac{33}{65}\Rightarrow\sin z=\frac{56}{65}$

$\therefore\tan z=\frac{56}{33}\Rightarrow z=\tan^{-1}\frac{56}{33}$

$\therefore\cos^{-1}\frac{33}{65}=\tan^{-1}\frac{56}{33}$ ....(3)

Now, we will prove that:

L.H.S = $\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}$

$=\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{5}{12}$ [Using 1 and 2]

= $\tan^{-1}\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\cdot\frac{5}{12}}$ $\left[\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}\right]$

$=\tan^{-1}\frac{36+20}{48-15}$

$=\tan^{-1}\frac{56}{33}$

$=\tan^{-1}\frac{56}{33}$ [by(3)]

= R.H.S

Question6. Prove that:

$$\cos^{-1}\frac{12}{13}+\sin^{-1}\frac{3}{5}=\sin^{-1}\frac{56}{65}$$

Solution

Let sin^(-1) 3/5 = x. Then $\sin x=\frac{3}{5} \Rightarrow \cos x=\sqrt{1-\left(\frac{3}{5}\right)^{2}}=\sqrt{\frac{16}{25}}=\frac{4}{5}$

$\therefore \tan x=\frac{3}{4} \Rightarrow x=\tan^{-1}\frac{3}{4}$

$\therefore \sin^{-1}\frac{3}{5}=\tan^{-1}\frac{3}{4}$ ....(1)

Now let $\cos^{-1}\frac{12}{13}=y$. Then $\cos y=\frac{12}{13} \Rightarrow \sin y=\frac{5}{13}$

$\therefore \tan y=\frac{5}{12} \Rightarrow y=\tan^{-1}\frac{5}{12}$

$\therefore \cos^{-1}\frac{12}{13}=\tan^{-1}\frac{5}{12}$ ...(2)

Let $\sin^{-1}\frac{56}{65}=z$ Then $\sin z=\frac{56}{65} \Rightarrow \cos z=\frac{33}{65}$

$\therefore \tan z=\frac{56}{33} \Rightarrow z=\tan^{-1}\frac{56}{33}$

$\therefore \sin^{-1}\frac{56}{65}=\tan^{-1}\frac{56}{33}$ ..(3)

Now, we have:

L.H.S = $\cos^{-1}\frac{12}{13}+\sin^{-1}\frac{3}{5}$

$=\tan^{-1}\frac{5}{12}+\tan^{-1}\frac{3}{4}$ [Using 1 and 2]

$=\tan^{-1}\frac{\frac{5}{12}+\frac{3}{4}}{1-\frac{5}{12}\cdot\frac{3}{4}}$ $\left[\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}\right]$

$=\tan^{-1}\frac{20+36}{48-15}$

$=\tan^{-1}\frac{56}{33}$

$=\sin^{-1}\frac{56}{65}$ = R.H.S [Using (3)]

Question7. Prove that: $\tan^{-1}\frac{63}{16}=\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}$

Solution

Let sin^(-1) 5/13 = x. Then, $\sin x=\frac{5}{13}\Rightarrow\cos x=\frac{12}{13}$

$\therefore\tan x=\frac{5}{12}\Rightarrow x=\tan^{-1}\frac{5}{12}$

$\therefore\sin^{-1}\frac{5}{13}=\tan^{-1}\frac{5}{12}$ ..(1)

Let $\frac{\cos^{-1}3}{5}=y$. Then $\cos y=\frac{3}{5}\Rightarrow\sin y=\frac{4}{5}$

$\therefore\tan y=\frac{4}{3}\Rightarrow y=\tan^{-1}\frac{4}{3}$

$\therefore\frac{\cos^{-1}3}{5}=\tan^{-1}\frac{4}{3}$ ...(2)

Using (1) and (2), we have

R.H.S = $\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}$

$=\tan^{-1}\frac{5}{12}+\cos^{-1}\frac{4}{5}$

$=\tan^{-1}\frac{5}{12}+\tan^{-1}\frac{4}{3}$

$=\tan^{-1}\left(\frac{\frac{5}{12}+\frac{4}{3}}{1-\frac{5}{12}\times\frac{4}{3}}\right)$ $\left[\tan^{-1}x+\tan^{-1}y=\frac{\tan^{-1}(x+y)}{1-xy}\right]$

$=\tan^{-1}\left(\frac{15+48}{36-20}\right)$

$=\tan^{-1}\frac{63}{16}$

= L.H.S

Question8. Prove that: $\tan^{-1}\sqrt{x}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right),x\in[0,1]$

Solution

Let x = $\tan^2\theta$ Then $\sqrt{x}=\tan\theta\Rightarrow\ \theta=\tan^{-1}\sqrt{x}$

$\therefore\frac{1-x}{1+x}=\frac{1-\tan^2\theta}{1+\tan^2\theta}=\cos 2\theta$

Now we have

R.H.S = $\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\cos^{-1}(\cos 2\theta)=\frac{1}{2}\times 2\theta=\theta=\ \tan^{-1}\sqrt{x}$ = L.H.S

Prove that 9: $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2}, x\in\left(0,\frac{\pi}{4}\right)$

Solution

Consider $\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2}\ x\in\left(0,\frac{\pi}{4}\right)$

$=\frac{\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)^2}{\left(\sqrt{1+\sin x}\right)^2-\left(\sqrt{1-\sin x}\right)^2}$ (by rationalizing)

$=\frac{(1+\sin x)+(1-\sin x)+2\sqrt{(1+\sin x)(1-\sin x)}}{1+\sin x-1+\sin x}$

$=\frac{2\left(1+\sqrt{1-\sin^2 x}\right)}{2\sin x}=\frac{1+\cos x}{\sin x}=\frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}$

$=\frac{\cot x}{2}$

∴ L.H.S = $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\cot^{-1}\left(\frac{\cot x}{2}\right)=\frac{x}{2}=R.H.S$

Question 10.Provethat:

$\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x.-\frac{1}{\sqrt{2}}\leq x\leq 1$ [Hint: put x = $\cos 2\theta$]

Put x = $\cos 2\theta$ so that $\theta = \frac{1}{2}\cos^{-1} x$ Then we have

L.H.S $= \tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$

$= \tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$

$= \tan^{-1}\left(\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}\right)$

$\tan^{-1}\left(\frac{\sqrt{2}\cos\theta-\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}\right)$

$= \tan^{-1}\left(\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right) = \tan^{-1}\left(\frac{1-\tan^{\theta}}{1+\tan\theta}\right)$

$= \tan^{-1}1 - \tan^{-1}(\tan\theta) \qquad \left[\tan^{-1}\left(\frac{x-y}{1+xy}\right) = \tan^{-1}x - \tan^{-1}y\right]$

$= \frac{\pi}{4} - \theta = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$ =R.H.S

Question12. Prove that: $\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3} = \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$

Solution

L.H.S $= \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3}$

$= \frac{9}{4}\left(\frac{\pi}{2} - \sin^{-1}\frac{1}{3}\right)$

$= \frac{9}{4}\left(\cos^{-1}\frac{1}{3}\right)$ ....(1) $\left[\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$

Now, let $\cos^{-1}\frac{1}{3} = x$ Then, $\cos x = \frac{1}{3} \Rightarrow \sin x = \sqrt{1-\left(\frac{1}{3}\right)^2} = \frac{2\sqrt{2}}{3}$

$\therefore x = \frac{\sin^{-1}\left(2\sqrt{2}\right)}{3} \Rightarrow \cos^{-1}\frac{1}{3} = \sin^{-1}\frac{2\sqrt{2}}{3}$

∴ L.H.S = $\frac{9}{4}\sin^{-1}\frac{2\left(\sqrt{2}\right)}{3}$ = R.H.S

Question13. Solve the equation: $2\tan^{-1}(\cos x) = \tan^{-1}(2\,\mathrm{cosec}\,x)$

Solution

$2\tan^{-1}(\cos x) = \tan^{-1}(2\cos ecx)$

$\Rightarrow \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{1}(2\cos ecx) \quad \left[2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x}\right]$

$\Rightarrow \frac{2\cos x}{1-\cos^2 x} = 2\cos ecx$

$\Rightarrow \frac{2\cos x}{\sin^2 x} = \frac{2}{\sin x}$

=> cos x = sin x

=> tan x = 1

$\therefore x = \frac{\pi}{4}$

Question13.

Solve sin ($\tan^{-1}$ x), | x| < 1 is equal to

(A) $\frac{x}{\sqrt{1-x^2}}$

(**B**) $\frac{1}{\sqrt{1-x^2}}$

(**C**) $\frac{1}{\sqrt{1+x^2}}$

(**D**) $\frac{x}{\sqrt{1+x^2}}$

Solution :

Let $\tan^{-1} x = y$. Then, tan y = x => $\sin y = \frac{x}{\sqrt{1+x}}$

$\therefore y = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) \Rightarrow \tan^{-1}x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$

$\therefore \sin(\tan^{-1}x) = \sin\left(\sin^{-1}\frac{x}{\sqrt{1+x^2}}\right) = \frac{x}{\sqrt{1+x^2}}$

Therefore, option (D) is correct.

Question14.

Solve $\sin^{-1}(1 - x) - 2\sin^{-1} x = \frac{\pi}{2}$, then x is equal to

(A) $0, \frac{1}{2}$

(B) $1, \frac{1}{2}$

(C) 0

(D) $\frac{1}{2}$

Solution

$\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$

$\Rightarrow -2\sin^{-1}x = \frac{\pi}{2} - \sin^{-1}(1-x)$

$\Rightarrow -2\sin^{-1}x = \cos^{-1}(1-x)$....(1)

Let $\sin^{-1}x = \theta \Rightarrow = x \Rightarrow \cos\theta = \sqrt{1-x^2}$

$\therefore \theta = \cos^{-1}\left(\sqrt{1-x^2}\right)$

$\therefore \sin^{-1}x = \cos^{-1}\left(\sqrt{1-x^2}\right)$

Therefore, from equation (1), we have

$-2\cos^{-1}\left(\sqrt{1-x^2}\right) = \cos^{-1}(1-x)$

Put $x = \sin y$. Then, we have:

$-2\cos^{-1}\left(\sqrt{1-\sin^2 y}\right) = \cos^{-1}(1-\sin y)$

$\Rightarrow -2\cos^{-1}(\cos y) = \cos^{-1}(1-\sin y)$

$\Rightarrow -2y = \cos^{-1}(1-\sin y)$

$\Rightarrow 1 - \sin y = \cos(-2y) = \cos 2y$

$\Rightarrow 1 - \sin y = 1 - 2\sin^2 y$

$\Rightarrow 2\sin^2 y - \sin y = 0$

$\Rightarrow \sin y(2\sin y - 1) = 0$

$\Rightarrow \sin y(2\sin y - 1) = 0$

$\Rightarrow \sin y = 0$ or $\frac{1}{2}$

$\therefore x = 0$ or $x = \frac{1}{2}$

But, when $x = \frac{1}{2}$, it can be observed that:

L.H.S = $\sin^{-1}\left(1 - \frac{1}{2}\right) - 2\sin^{-1}\frac{1}{2}$

$= \sin^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\frac{1}{2}$

$= \sin^{-1}\frac{1}{2}$

= $-\frac{\pi}{6} \neq \frac{\pi}{2} \neq$ R.H.S

∴ x = 1/2 is not the solution of the given equation.

Thus, x = 0.

Therefore, option (C) is correct.