

. Exercise 7.1 Page: 105

1. Find the distance between the following pairs of points:

(i) (2, 3), (4,1)

(ii) (-5, 7), (-1, 3)

(iii) (a, b), (-a, -b)

Answer:

(i) Distance between the points is given by

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Therefore the distance between (2,3) and (4,1) is given by

$$l = \sqrt{(2 - 4)^2 + (3 - 1)^2}$$

$$= \sqrt{(-2)^2 + (2)^2}$$

$$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

(ii)Applying Distance Formula to find distance between points (-5, 7) and (-1, 3), we get

$$l = \sqrt{(-5 - (-1))^2 + (7 - 3)^2}$$

$$= \sqrt{(-4)^2 + (4)^2}$$

$$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

(iii) Applying Distance Formula to find distance between points (a, b) and (-a, -b), we get

$$\begin{aligned} | &= \sqrt{(a - (-a))^2 + (b - (-b))^2} \\ &= \sqrt{(2a)^2 + (2b)^2} \\ &= \sqrt{4a^2 + 4b^2} = 2\sqrt{a^2 + b^2} \end{aligned}$$

2. Find the distance between the points (0, 0) and (36, 15). Also, find the distance between towns A and B if town B is located at 36 km east and 15 km north of town A.

Answer:

Applying Distance Formula to find distance between points (0, 0) and (36, 15), we get

$$\begin{aligned} &= \sqrt{(36 - 0)^2 + (15 - 0)^2} = \sqrt{36^2 + 15^2} \\ &= \sqrt{1296 + 225} = \sqrt{1521} = 39 \end{aligned}$$

Yes, we can find the distance between the given towns A and B.

Assume town A at origin point (0, 0).

Therefore, town B will be at point (36, 15) with respect to town A.

And hence, as calculated above, the distance between town A and B will be 39km.

3. Determine if the points (1, 5), (2, 3) and (-2, -11) are collinear.

Answer:

Let A = (1, 5), B = (2, 3) and C = (-2, -11)

Using Distance Formula to find distance AB, BC and CA.

$$AB = \sqrt{(1 - 2)^2 + (5 - 3)^2} = \sqrt{5}$$

$$BC = \sqrt{(2 - (-2))^2 + (3 - (-11))^2} = \sqrt{4^2 + 14^2} = \sqrt{16 + 196} = \sqrt{212}$$

$$CA = \sqrt{(1 - (-2))^2 + (5 - (-11))^2} = \sqrt{3^2 + 16^2} = \sqrt{9 + 256} = \sqrt{265}$$

Since $AB + BC \neq CA$

Therefore, the points (1, 5), (2, 3), and (-2, -11) are not collinear.

4. Check whether (5, -2), (6, 4) and (7, -2) are the vertices of an isosceles triangle.

Answer:

Let A = (5, -2), B = (6, 4) and C = (7, -2)

Using Distance Formula to find distances AB, BC and CA.

$$AB = \sqrt{(5 - 6)^2 + (-2 - 4)^2} = \sqrt{(-1)^2 + (-6)^2} = \sqrt{1 + 36} = \sqrt{37}$$

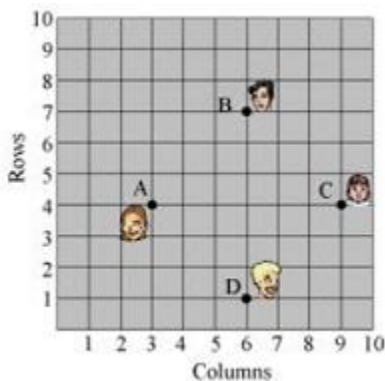
$$BC = \sqrt{(6 - 7)^2 + (4 - (-2))^2} = \sqrt{(-1)^2 + (6)^2} = \sqrt{1 + 36} = \sqrt{37}$$

$$CA = \sqrt{(5 - 7)^2 + (-2 - (-2))^2} = \sqrt{(-2)^2 + 0^2} = 2$$

Since $AB = BC$.

Therefore, A, B and C are vertices of an isosceles triangle.

5. In a classroom, 4 friends are seated at the points A (3, 4), B (6, 7), C (9, 4) and D (6, 1). Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli. “Don’t you think ABCD is a square?” Chameli disagrees. Using distance formula, find which of them is correct.



Answer:

We have $A = (3, 4)$, $B = (6, 7)$, $C = (9, 4)$ and $D = (6, 1)$

Using Distance Formula to find distances AB, BC, CD and DA, we get

$$AB = \sqrt{(3 - 6)^2 + (4 - 7)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$$

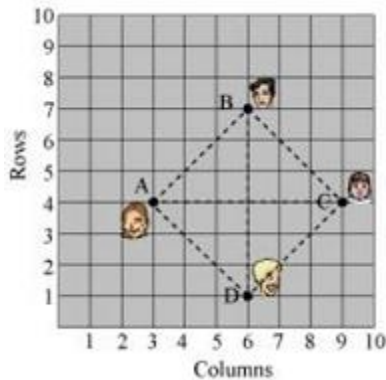
$$BC = \sqrt{(6 - 9)^2 + (7 - 4)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$$

$$CD = \sqrt{(9 - 6)^2 + (4 - 1)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$$

$$AD = \sqrt{(3 - 6)^2 + (4 - 1)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$$

Therefore, All the sides of ABCD are equal here

Now, we will check the length of its diagonals.



$$AC = \sqrt{(3 - 9)^2 + (4 - 4)^2} = \sqrt{(-6)^2} = 6$$

$$BD = \sqrt{(6 - 6)^2 + (7 - 1)^2} = \sqrt{(6)^2} = 6$$

So, Diagonals of ABCD are also equal.

we can definitely say that ABCD is a square.

Therefore, Champa is correct.

6. Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer.

(i) $(-1, -2), (1, 0), (-1, 2), (-3, 0)$

(ii) $(-3, 5), (3, 1), (0, 3), (-1, -4)$

(iii) $(4, 5), (7, 6), (4, 3), (1, 2)$

Answer:

(i) Let $A = (-1, -2), B = (1, 0), C = (-1, 2)$ and $D = (-3, 0)$

Using Distance Formula to find distances AB, BC, CD and DA, we get

$$AB = \sqrt{(-1 - 1)^2 + (-2 - 0)^2} = \sqrt{(-2)^2 + (-2)^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$$

$$BC = \sqrt{(1 - (-1))^2 + (0 - 2)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$$

$$CD = \sqrt{(-1 - (-3))^2 + (2 - 0)^2} = \sqrt{(2)^2 + (2)^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$$

$$AD = \sqrt{(-1 - (3))^2 + (-2 - 0)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$$

Therefore, all four sides of quadrilateral are equal.

Now, we will check the length of diagonals.

$$AC = \sqrt{(-1 - (-1))^2 + (-2 - 2)^2} = \sqrt{0^2 + (-4)^2} = \sqrt{16} = 4$$

$$BD = \sqrt{(1 - (-3))^2 + (0 - 0)^2} = \sqrt{(4)^2 + 0^2} = \sqrt{16} = 4$$

Therefore, diagonals of quadrilateral ABCD are also equal.

we can say that ABCD is a square.

(ii) Let A = (-3, 5), B = (3, 1), C = (0, 3) and D = (-1, -4)

Using Distance Formula to find distances AB, BC, CD and DA, we get

$$AB = \sqrt{(-3 - 3)^2 + (5 - 1)^2} = \sqrt{(-6)^2 + (4)^2} = \sqrt{36 + 16} = \sqrt{52} = 2\sqrt{13}$$

$$BC = \sqrt{(3 - 0)^2 + (1 - 3)^2} = \sqrt{(3)^2 + (-2)^2} = \sqrt{9 + 4} = \sqrt{13}$$

$$CD = \sqrt{(0 - (-1))^2 + (3 - (-4))^2} = \sqrt{(1)^2 + (7)^2} = \sqrt{1 + 49} = \sqrt{50} = 5\sqrt{2}$$

$$DA = \sqrt{(-3 - (-1))^2 + (5 - (-4))^2} = \sqrt{(-2)^2 + (9)^2} = \sqrt{4 + 81} = \sqrt{85}$$

We cannot find any relation between the lengths of different sides.

Therefore, we cannot give any name to the quadrilateral ABCD.

(iii) Let $A = (4, 5)$, $B = (7, 6)$, $C = (4, 3)$ and $D = (1, 2)$

Using Distance Formula to find distances AB, BC, CD and DA, we get

$$AB = \sqrt{(4-7)^2 + (5-6)^2} = \sqrt{(-3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$$

$$BC = \sqrt{(7-4)^2 + (6-3)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18}$$

$$CD = \sqrt{(4-1)^2 + (3-2)^2} = \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10}$$

$$DA = \sqrt{(4-1)^2 + (5-2)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18}$$

Here opposite sides of quadrilateral ABCD are equal.

We can now find out the lengths of diagonals.

$$AC = \sqrt{(4-4)^2 + (5-3)^2} = \sqrt{(0)^2 + (2)^2} = \sqrt{0+4} = 2$$

$$BD = \sqrt{(7-1)^2 + (6-2)^2} = \sqrt{(6)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52} = 13\sqrt{2}$$

Here diagonals of ABCD are not equal.

we can say that ABCD is not a rectangle therefore it is a parallelogram.

7. Find the point on the x-axis which is equidistant from (2, -5) and (-2, 9).

Answer:

Let the point be $(x, 0)$ on x-axis which is equidistant from $(2, -5)$ and $(-2, 9)$.

Using Distance Formula and according to given conditions we have:

$$\sqrt{(x-2)^2 + (0-(-5))^2} = \sqrt{(x-2)^2 + (5)^2}$$

$$\Rightarrow \sqrt{(x-(-2))^2 + (0-(-9))^2} = \sqrt{(x+2)^2 + (9)^2}$$

Squaring both sides, we get

$$\Rightarrow \sqrt{(x-2)^2 + (5)^2} = \sqrt{(x+2)^2 + (9)^2}$$

$$(x-2)^2 + 25 = (x+2)^2 + 81$$

$$x^2 + 4 - 4x + 25 = x^2 + 4 + 4x + 81$$

$$8x = -25 - 81$$

$$8x = -56$$

$$x = -7$$

Therefore, point on the x-axis which is equidistant from $(2, -5)$ and $(-2, 9)$ is $(-7, 0)$

8. Find the values of y for which the distance between the points $P(2, -3)$ and $Q(10, y)$ is 10 units.

Answer:

Using Distance formula, we have

$$\sqrt{(2-10)^2 + (-3-y)^2} = 10$$

$$\Rightarrow \sqrt{(-8)^2 + (3 + y)^2} = 10$$

$$\Rightarrow 64 + (y + 3)^2 = 100$$

$$\Rightarrow (y+3)^2 = 100-64 = 36$$

$$\Rightarrow y+3 = \pm 6$$

$$\Rightarrow y+3=6 \text{ or } y+3 = - 6$$

Therefore $y = 3$ or -9

9. If, Q (0, 1) is equidistant from P (5, -3) and R (x, 6), find the values of x. Also, find the distances QR and PR.

Answer:

It is given that Q is equidistant from P and R. Using Distance Formula, we get

$$PQ = RQ$$

$$\sqrt{(5 - 0)^2 + (-3 - 1)^2} = \sqrt{(0 - x)^2 + (1 - 6)^2}$$

$$\sqrt{(5)^2 + (-4)^2} = \sqrt{(-x)^2 + (-5)^2}$$

$$\Rightarrow \sqrt{25+16} = \sqrt{x^2 + 25}$$

$$\Rightarrow 41 = x^2 + 25$$

$$16 = x^2$$

$$x = \pm 4$$

Thus, R is (4, 6) or (-4, 6).

When point R is (4,6)

$$PR = \sqrt{(5-4)^2 + (-3-6)^2} = \sqrt{(1^2 + (-9)^2)} = \sqrt{1+81} = \sqrt{82}$$

$$QR = \sqrt{(0-4)^2 + (1-6)^2} = \sqrt{(-4)^2 + (-5)^2} = \sqrt{16+25} = \sqrt{41}$$

When point R is (-4,6)

$$PR = \sqrt{(5-(-4))^2 + (-3-6)^2} = \sqrt{(9)^2 + (-9)^2} = \sqrt{81+81} = 9\sqrt{2}$$

$$QR = \sqrt{(0-(-4))^2 + (1-6)^2} = \sqrt{(4)^2 + (-5)^2} = \sqrt{16+25} = \sqrt{41}$$

10. Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (-3, 4).

Answer:

It is given that (x, y) is equidistant from (3, 6) and (-3, 4).

Using Distance formula, we can write

$$\sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x-(-3))^2 + (y-4)^2}$$

$$\Rightarrow \sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x+3)^2 + (y-4)^2}$$

$$\Rightarrow (x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$$

$$\Rightarrow x^2 + 9 - 6x + y^2 + 36 - 12y = x^2 + 9 + 6x + y^2 + 16 - 8y$$

$$\Rightarrow 36 - 16 = 6x + 6x + 12y - 8y$$

$$\Rightarrow 20 = 12x + 4y$$

$$\Rightarrow 3x + y = 5$$

$$\Rightarrow 3x + y - 5 = 0$$

. Exercise 7.2 Page: 111

Solve the followings Questions.

1. Find the coordinates of the point which divides the join of (- 1, 7) and (4, - 3) in the ratio 2:3.

Answer:

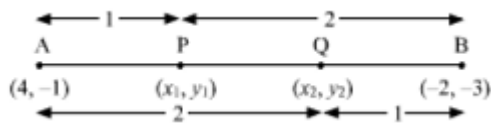
Let P(x, y) be the required point. Using the section formula

$$x = \frac{2 \times 4 + 3 \times (-1)}{2 + 3} = \frac{8 - 3}{5} = \frac{5}{5} = 1$$
$$y = \frac{2 \times (-3) + 3 \times 7}{2 + 3} = \frac{-6 + 21}{5} = \frac{15}{5} = 3$$

Therefore the point is (1,3).

2. Find the coordinates of the points of trisection of the line segment joining (4, -1) and (-2, -3).

Answer:



Let P (x_1, y_1) and Q (x_2, y_2) are the points of trisection of the line segment joining the given points i.e., AP = PQ = QB

Therefore, point P divides AB internally in the ratio 1:2.

$$x_1 = \frac{1 \times (-2) + 2 \times 4}{1 + 2}, y_1 = \frac{1 \times (-3) + 2 \times (-1)}{1 + 2}$$
$$x_1 = \frac{-2 + 8}{3} = \frac{6}{3} = 2, y_1 = \frac{-3 - 2}{3} = \frac{-5}{3}$$

Therefore P(x_1, y_1) = (2, -5/3)

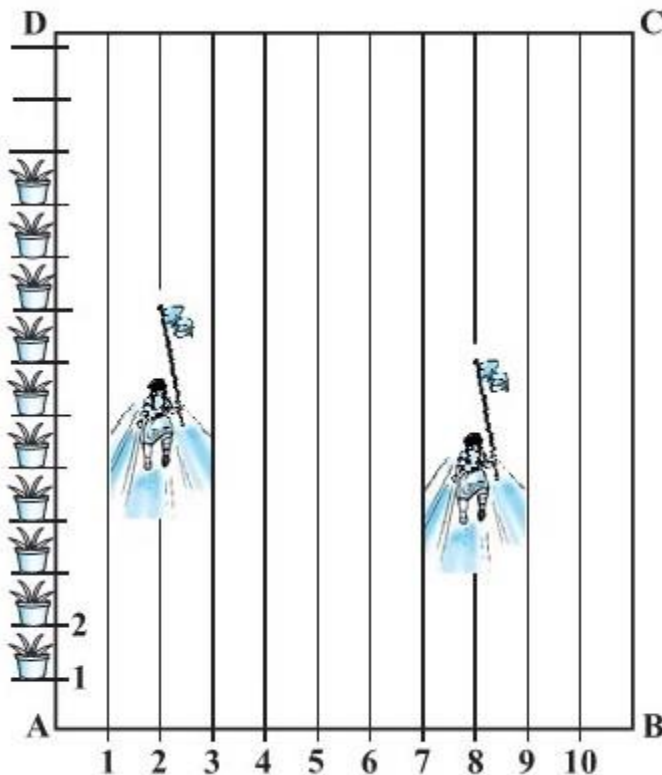
Point Q divides AB internally in the ratio 2:1.

$$x_2 = \frac{2 \times (-2) + 1 \times 4}{2 + 1}, y_2 = \frac{2 \times (-3) + 1 \times (-1)}{2 + 1}$$

$$x_2 = \frac{-4 + 4}{3} = 0, y_2 = \frac{-6 - 1}{3} = \frac{-7}{3}$$

$$Q(x_2, y_2) = (0, -7/3)$$

3. To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in the following figure. Niharika runs $1/4$ th the distance AD on the 2nd line and posts a green flag. Preet runs $1/5$ th the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?



Answer:

From the figure, taking A as (0, 0), x- axis along AB and y- axis along AD, we will obtain the coordinates of the green flag and the red flag.

$$\begin{aligned}\text{The green flag is at } & \frac{1}{4}\text{th of the total distance} \\ & = \frac{1}{4} \times 100 = 25 \text{ m in 2nd line.}\end{aligned}$$

∴ The coordinates of green flag are (2, 25).

Similarly, coordinates of red flag are (8, 20).

Distance between two flags,

$$\begin{aligned}D &= \sqrt{(8-2)^2 + (20-25)^2} \\ &= \sqrt{(6)^2 + (-5)^2} = \sqrt{36+25} = \sqrt{61} \text{ m.}\end{aligned}$$

Now, blue flag is posted at the midpoint of the distance between two flags

$$\begin{aligned}\therefore \text{Coordinates of blue flag} &= \left(\frac{2+8}{2}, \frac{25+20}{2} \right) \\ &= (5, 22.5)\end{aligned}$$

Hence, the blue flag will be posted in 5th line at a distance of **22.5 m**.

4. Find the ratio in which the line segment joining the points (-3, 10) and (6, - 8) is divided by (-1, 6).

Answer:

Let the ratio in which the line segment joining (-3, 10) and (6, -8) is divided by point (-1, 6) be k:1.

$$\text{Therefore, } -1 = \frac{6k-3}{k+1}$$

$$-k - 1 = 6k - 3$$

$$7k = 2$$

$$k = \frac{2}{7}$$

Therefore, the required ratio is 2:7.

5. Find the ratio in which the line segment joining A (1, - 5) and B (- 4, 5) is divided by the x-axis. Also find the coordinates of the point of division.

Answer:

Let the ratio in which the line segment joining A (1, - 5) and B (- 4, 5) is divided by x-axis be k:1.

Therefore, the coordinates of the point of division is $(\frac{-4k+1}{k+1}, \frac{5k-5}{k+1})$.

We know that y-coordinate of any point on x-axis is 0.

$$\therefore \frac{5k-5}{k+1} = 0$$

Therefore, x-axis divides it in the ratio 1:1.

To find the coordinates let's substitute the value of k in equation(1)

$$\text{Required point} = [(-4(1) + 1) / (1 + 1), (5(1) - 5) / (1 + 1)]$$

$$= [(-4 + 1) / 2, (5 - 5) / 2]$$

$$= [-3/2, 0]$$

6. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

Answer:

Let A,B,C and D be the points (1,2) (4,y), (x,6) and (3,5) respectively.

Mid-point of AC = Mid-point of BD

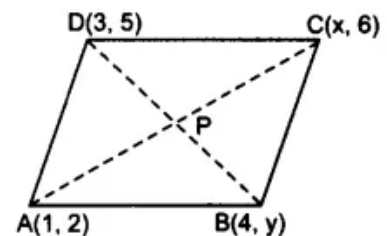
$$\Rightarrow \frac{x+1}{2}, \frac{6+2}{2} = \frac{4+3}{2}, \frac{y+5}{2}$$

$$\Rightarrow \frac{x+1}{2} = \frac{7}{2} \text{ and } \frac{6+2}{2} = \frac{y+5}{2}$$

$$\Rightarrow x + 1 = 7 \text{ and } 8 = y + 5$$

$$\Rightarrow x = 7 - 1 \text{ and } y = 8 - 5 = 3$$

$$\Rightarrow x = 6 \text{ and } y = 3$$



7. Find the coordinates of a point A, where AB is the diameter of circle whose centre is (2, -3) and B is (1, 4).

Answer:

Let (x,y) be the coordinate of A.

Since AB is the diameter of the circle, the centre will be the mid-point of AB.

now, as centre is the mid-point of AB.

$$\text{x-coordinate of centre} = (2x+1)/2$$

$$y\text{-coordinate of centre} = (2y+4)/2$$

But given that centre of circle is $(2, -3)$.

Therefore,

$$(2x+1)/2=2 \Rightarrow x=3$$

$$(2y+4)/2=-3 \Rightarrow y=-10$$

Thus the coordinate of A is $(3, -10)$.

8. If A and B are $(-2, -2)$ and $(2, -4)$, respectively, find the coordinates of P such that $AP = 3/7 AB$ and P lies on the line segment AB.

Answer:

As given the coordinates of A $(-2, -2)$ and B $(2, -4)$ and P is a point lies on AB.

$$\text{And } AP = 3/7 AB$$

$$\therefore BP = 4/7$$

Then, ratio of AP and PB = $m_1:m_2 = 3:4$

Let the coordinates of P be (x, y) .

$$\therefore x = (m_1x_2 + m_2x_1) / (m_1 + m_2)$$

$$\Rightarrow x = (3 \times 2 + 4 \times (-2)) / (3 + 4) = (6 - 8) / 7 = -2 / 7$$

$$\text{And } y = (m_1y_2 + m_2y_1) / (m_1 + m_2)$$

$$\Rightarrow y = ((3 \times (-4) + 4 \times (-2)) / (3 + 4) = (-12 - 8) / 7 = -20 / 7$$

\therefore Coordinates of P = $-2 / 7, -20 / 7$

9. Find the coordinates of the points which divide the line segment joining A $(-2, 2)$ and B $(2, 8)$ into four equal parts.

Answer:



From the figure, it can be observed that points X,Y,Z are dividing the line segment in a ratio 1:3,1:1,3:1 respectively.

Using Sectional Formula, we get,

$$\text{Coordinates of X} = ((1 \times 2 + 3 \times (-2)) / (1 + 3), (1 \times 8 + 3 \times 2) / (1 + 3))$$

$$= (-1, 7/2)$$

$$\text{Coordinates of Y} = (2 - 2) / 2, (2 + 8) / 2 = (0,5)$$

$$\text{Coordinates of Z} = ((3 \times 2 + 1 \times (-2)) / (1 + 3), (3 \times 8 + 1 \times 2) / (1 + 3))$$

$$=(1, 13/2)$$

10. Find the area of a rhombus if its vertices are (3, 0), (4, 5), (-1, 4) and (-2,-1) taken in order. [Hint: Area of a rhombus = 1/2(product of its diagonals)]

Answer:

Let (3, 0), (4, 5), (-1, 4) and (-2, -1) are the vertices A, B, C, D of a rhombus ABCD.

Length of the diagonal AC=

$$\sqrt{(3 - (-1))^2 + (0 - 4)^2} = \sqrt{16 + 16} = 4\sqrt{2}$$

Length of the diagonal BD=

$$\sqrt{(4 - (-2))^2 + (5 - (-1))^2} = \sqrt{36 + 36} = 6\sqrt{2}$$

Area of rhombus ABCD = 1/2 X $4\sqrt{2}$ X $6\sqrt{2}$ = 24 square units.

Therefore, the area of a rhombus if its vertices are (3, 0), (4, 5), (-1, 4) and (-2,-1) taken in order, is 24 square units.