Chapter 7 – Triangles

Exercise: 7.1 **Page No: 92**

1. In quadrilateral ACBD, AC = AD and AB bisect ∠A (see Fig. 7.16). Show that ΔABC≅ ΔABD. What can you say about BC and BD?

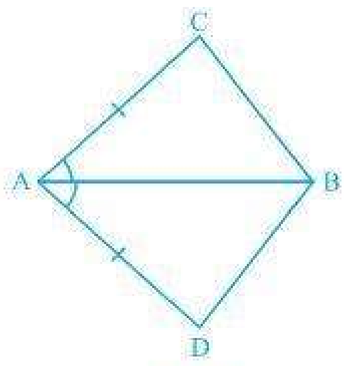

Fig. 7.16

Solution:

It is given that AC and AD are equal i.e. AC = AD and the line segment AB bisects ∠A.

We will have to now prove that the two triangles ABC and ABD are similar i.e. **ΔABC ≅ ΔABD**

Proof:

Consider the triangles ΔABC and ΔABD,

(i) AC = AD (It is given in the question)

(ii) AB = AB (Common)

(iii) ∠CAB = ∠DAB (Since AB is the bisector of angle A)

So, by **SAS congruency criterion**, ΔABC ≅ ΔABD.

For the 2nd part of the question, BC and BD are of equal lengths by the rule of C.P.C.T.

2. ABCD is a quadrilateral in which AD = BC and ∠DAB = ∠CBA (see Fig. 7.17). Prove that

(i) ΔABD ≅ ΔBAC

(ii) BD = AC

(iii) ∠ABD = ∠BAC.

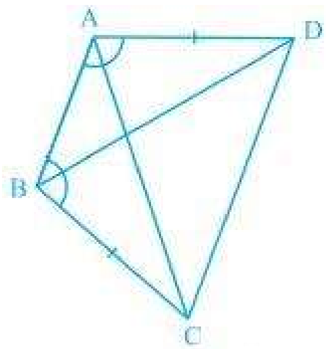

Fig. 7.17

Solution:

The given parameters from the questions are ∠DAB = ∠CBA and AD = BC.

(i) ΔABD and ΔBAC are similar by SAS congruency as

AB = BA (It is the common arm)

∠DAB = ∠CBA and AD = BC (These are given in the question)

So, triangles ABD and BAC are similar i.e. ΔABD ≅ ΔBAC. (Hence proved).

(ii) It is now known that ΔABD ≅ ΔBAC so,

BD = AC (by the rule of CPCT).

(iii) Since ΔABD ≅ ΔBAC so,

Angles ∠ABD = ∠BAC (by the rule of CPCT).

3. AD and BC are equal perpendiculars to a line segment AB (see Fig. 7.18). Show that CD bisects AB.

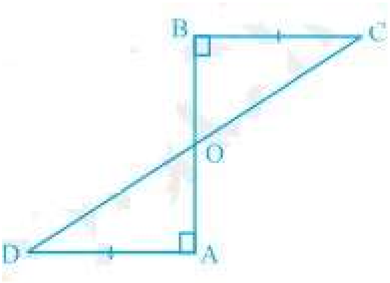

Fig. 7.18

Solution:

It is given that AD and BC are two equal perpendiculars to AB.

We will have to prove that **CD is the bisector of AB**

Now,

Triangles ΔAOD and ΔBOC are similar by AAS congruency since:

(i) ∠A = ∠B (They are perpendiculars)

(ii) AD = BC (As given in the question)

(iii) ∠AOD = ∠BOC (They are vertically opposite angles)

∴ ΔAOD ≅ ΔBOC.

So, AO = OB (by the rule of CPCT).

Thus, CD bisects AB (Hence proved).

4. *l* and *m* are two parallel lines intersected by another pair of parallel lines p and q (see Fig. 7.19). Show that ΔABC ≅ ΔCDA.

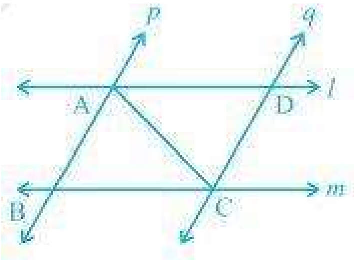

Fig. 7.19

Solution:

It is given that p || q and l || m

To prove:

Triangles ABC and CDA are similar i.e. ΔABC ≅ ΔCDA

Proof:

Consider the ΔABC and ΔCDA,

(i) ∠BCA = ∠DAC and ∠BAC = ∠DCA Since they are alternate interior angles

(ii) AC = CA as it is the common arm

So, by **ASA congruency criterion,** ΔABC ≅ ΔCDA.

5. Line l is the bisector of an angle ∠A and B is any point on *l*. BP and BQ are perpendiculars from B to the arms of ∠A (see Fig. 7.20). Show that:

(i) ΔAPB ≅ ΔAQB

(ii) BP = BQ or B is equidistant from the arms of ∠A.

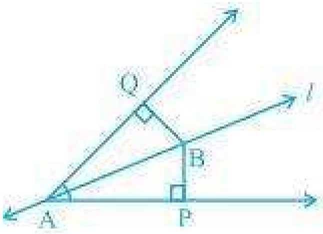

Fig. 7.20

Solution:

It is given that the line "*l*" is the bisector of angle ∠A and the line segments BP and BQ are perpendiculars drawn from *l*.

(i) ΔAPB and ΔAQB are similar by AAS congruency because:

∠P = ∠Q (They are the two right angles)

AB = AB (It is the common arm)

∠BAP = ∠BAQ (As line *l* is the bisector of angle A)

So, ΔAPB ≅ ΔAQB.

(ii) By the rule of CPCT, BP = BQ. So, it can be said the point B is equidistant from the arms of ∠A.

6. In Fig. 7.21, AC = AE, AB = AD and ∠BAD = ∠EAC. Show that BC = DE.

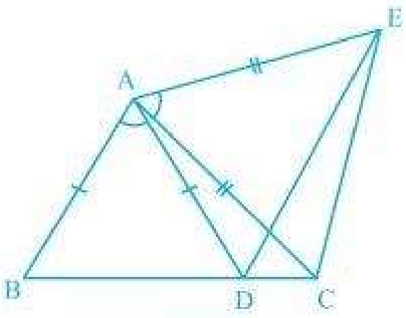

Fig. 7.21

Solution:

It is given in the question that AB = AD, AC = AE, and ∠BAD = ∠EAC

To prove:

The line segment BC and DE are similar i.e. BC = DE

Proof:

We know that ∠BAD = ∠EAC

Now, by adding ∠DAC on both sides we get,

∠BAD + ∠DAC = ∠EAC +∠DAC

This implies, ∠BAC = ∠EAD

Now, ΔABC and ΔADE are similar by SAS congruency since:

(i) AC = AE (As given in the question)

(ii) ∠BAC = ∠EAD

(iii) AB = AD (It is also given in the question)

∴ Triangles ABC and ADE are similar i.e. ΔABC ≅ ΔADE.

So, by the rule of CPCT, it can be said that BC = DE.

7. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that ∠BAD = ∠ABE and ∠EPA = ∠DPB (see Fig. 7.22). Show that

(i) ΔDAP ≅ ΔEBP

(ii) AD = BE

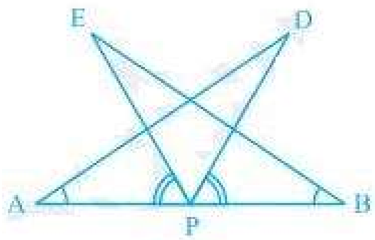

Fig. 7.22

Solutions:

In the question, it is given that P is the mid-point of line segment AB. Also, ∠BAD = ∠ABE and ∠EPA = ∠DPB

(i) It is given that ∠EPA = ∠DPB

Now, add ∠DPE on both sides,

∠EPA +∠DPE = ∠DPB+∠DPE

This implies that angles DPA and EPB are equal i.e. ∠DPA = ∠EPB

Now, consider the triangles DAP and EBP.

∠DPA = ∠EPB

AP = BP (Since P is the mid-point of the line segment AB)

∠BAD = ∠ABE (As given in the question)

So, by **ASA congruency**, ΔDAP ≅ ΔEBP.

(ii) By the rule of CPCT, AD = BE.

8. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see Fig. 7.23). Show that:

(i) ΔAMC ≅ ΔBMD

(ii) ∠DBC is a right angle.

(iii) ΔDBC ≅ ΔACB

(iv) CM = ½ AB

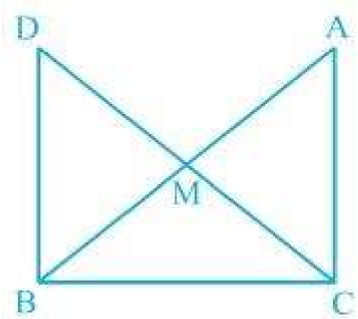

Fig. 7.23

Solution:

It is given that M is the mid-point of the line segment AB, ∠C = 90°, and DM = CM

(i) Consider the triangles ΔAMC and ΔBMD:

AM = BM (Since M is the mid-point)

CM = DM (Given in the question)

∠CMA = ∠DMB (They are vertically opposite angles)

So, by **SAS congruency criterion**, ΔAMC ≅ ΔBMD.

(ii) ∠ACM = ∠BDM (by CPCT)

∴ AC || BD as alternate interior angles are equal.

Now, ∠ACB +∠DBC = 180° (Since they are co-interiors angles)

⇒ 90° +∠B = 180°

∴ ∠DBC = 90°

(iii) In ΔDBC and ΔACB,

BC = CB (Common side)

∠ACB = ∠DBC (They are right angles)

DB = AC (by CPCT)

So, ΔDBC ≅ ΔACB by **SAS congruency**.

(iv) DC = AB (Since ΔDBC ≅ ΔACB)

⇒ DM = CM = AM = BM (Since M the is mid-point)

So, DM + CM = BM+AM

Hence, CM + CM = AB

$\Rightarrow$ CM = (½) AB