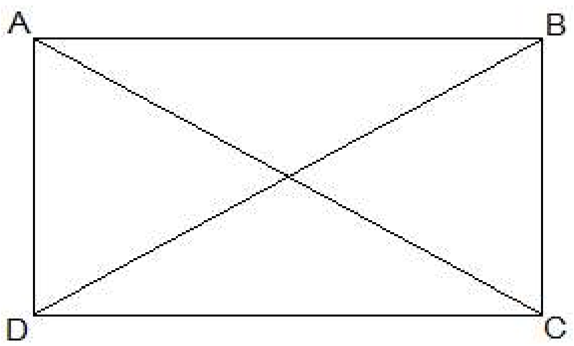
CHAPTER 8 QUADRILATERALS

EXERCISE 8.1 PAGE:110

1. If the diagonals of a parallelogram are equal, then show that it is a rectangle.

Solution:



Given that,

AC = BD

To show that ABCD is a rectangle if the diagonals of a parallelogram are equal

To show ABCD is a rectangle, we have to prove that one of its interior angles is right-angled.

Proof,

In ΔABC and ΔBAD,

AB = BA (Common)

BC = AD (Opposite sides of a parallelogram are equal)

AC = BD (Given)

Therefore, $\Delta ABC \cong \Delta BAD$ [SSS congruency]

$\angle A = \angle B$ [Corresponding parts of Congruent Triangles]

also,

$\angle A + \angle B = 180^\circ$ (Sum of the angles on the same side of the transversal)

$\Rightarrow 2\angle A = 180^\circ$

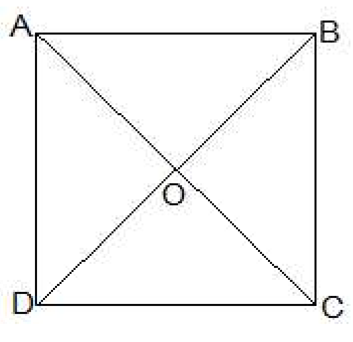
$\Rightarrow \angle A = 90^\circ = \angle B$

Therefore, ABCD is a rectangle.

Hence Proved.

2. Show that the diagonals of a square are equal and bisect each other at right angles.

Solution:



Let ABCD be a square and its diagonals AC and BD intersect each other at O.

To show that,

AC = BD

AO = OC

and ∠AOB = 90º

Proof,

In ΔABC and ΔBAD,

AB = BA (Common)

∠ABC = ∠BAD = 90º

BC = AD (Given)

ΔABC ≅ ΔBAD [SAS congruency]

Thus,

AC = BD [CPCT]

diagonals are equal.

Now,

In ΔAOB and ΔCOD,

∠BAO = ∠DCO (Alternate interior angles)

∠AOB = ∠COD (Vertically opposite)

AB = CD (Given)

, ΔAOB ≅ ΔCOD [AAS congruency]

Thus,

AO = CO [CPCT].

, Diagonal bisect each other.

Now,

In ΔAOB and ΔCOB,

OB = OB (Given)

AO = CO (diagonals are bisected)

AB = CB (Sides of the square)

, ΔAOB ≅ ΔCOB [SSS congruency]

also, ∠AOB = ∠COB

∠AOB+∠COB = 180° (Linear pair)

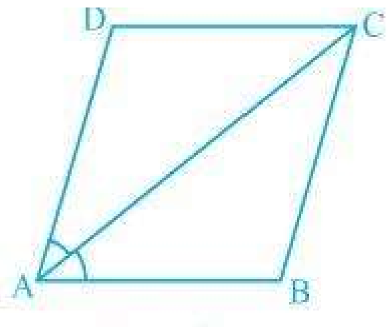
Thus, ∠AOB = ∠COB = 90°

, Diagonals bisect each other at right angles

3. Diagonal AC of a parallelogram ABCD bisects ∠A (see Fig. 8.11). Show that

(i) it bisects ∠C also,

(ii) ABCD is a rhombus.



Solution:

(i) In ΔADC and ΔCBA,

AD = CB (Opposite sides of a parallelogram)

DC = BA (Opposite sides of a parallelogram)

AC = CA (Common Side)

, ΔADC ≅ ΔCBA [SSS congruency]

Thus,

∠ACD = ∠CAB by CPCT

and ∠CAB = ∠CAD (Given)

⇒ ∠ACD = ∠BCA

Thus,

AC bisects ∠C also.

(ii) ∠ACD = ∠CAD (Proved above)

⇒ AD = CD (Opposite sides of equal angles of a triangle are equal)

Also, AB = BC = CD = DA (Opposite sides of a parallelogram)

Thus,

ABCD is a rhombus.

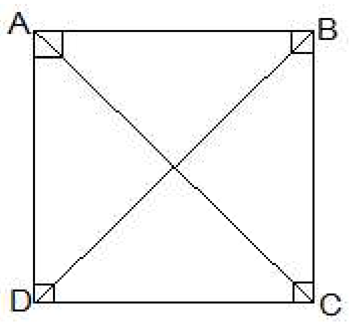
4. ABCD is a rectangle in which diagonal AC bisects ∠A as well as ∠C.

Show that:

(i) ABCD is a square

(ii) Diagonal BD bisects ∠B as well as ∠D.

Solution:



(i) ∠DAC = ∠DCA (AC bisects ∠A as well as ∠C)

⇒ AD = CD (Sides opposite to equal angles of a triangle are equal)

also, CD = AB (Opposite sides of a rectangle)

,AB = BC = CD = AD

Thus, ABCD is a square.

(ii) In ΔBCD,

BC = CD

⇒ ∠CDB = ∠CBD (Angles opposite to equal sides are equal)

also, ∠CDB = ∠ABD (Alternate interior angles)

⇒ ∠CBD = ∠ABD

Thus, BD bisects ∠B

Now,

∠CBD = ∠ADB

⇒ ∠CDB = ∠ADB

Thus, BD bisects ∠B as well as ∠D.

5. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (see Fig. 8.12). Show that:

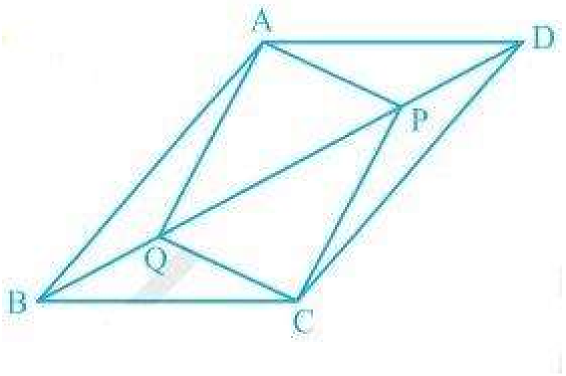
(i) ΔAPD ≅ ΔCQB

(ii) AP = CQ

(iii) ΔAQB ≅ ΔCPD

(iv) AQ = CP

(v) APCQ is a parallelogram



Solution:

(i) In ΔAPD and ΔCQB,

DP = BQ (Given)

∠ADP = ∠CBQ (Alternate interior angles)

AD = BC (Opposite sides of a parallelogram)

Thus, ΔAPD ≅ ΔCQB [SAS congruency]

(ii) AP = CQ by CPCT as ΔAPD ≅ ΔCQB.

(iii) In ΔAQB and ΔCPD,

BQ = DP (Given)

∠ABQ = ∠CDP (Alternate interior angles)

AB = CD (Opposite sides of a parallelogram)

Thus, ΔAQB ≅ ΔCPD [SAS congruency]

(iv) As ΔAQB ≅ ΔCPD

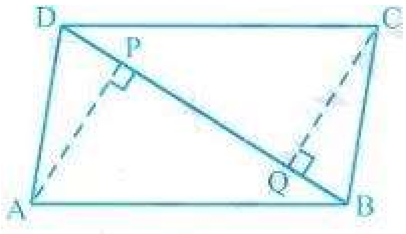
AQ = CP [CPCT]

(v) From the questions (ii) and (iv), it is clear that APCQ has equal opposite sides and also has equal and opposite angles. , APCQ is a parallelogram.

6. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see Fig. 8.13). Show that

(i) ΔAPB ≅ ΔCQD

(ii) AP = CQ



Solution:

(i) In ΔAPB and ΔCQD,

∠ABP = ∠CDQ (Alternate interior angles)

∠APB = ∠CQD (= 90° as AP and CQ are perpendiculars)

AB = CD (ABCD is a parallelogram)

, ΔAPB ≅ ΔCQD [AAS congruency]

(ii) As ΔAPB ≅ ΔCQD.

, AP = CQ [CPCT]

7. ABCD is a trapezium in which AB || CD and AD = BC (see Fig. 8.14). Show that

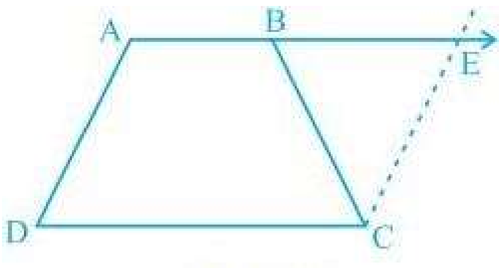
(i) ∠A = ∠B

(ii) ∠C = ∠D

(iii) ΔABC ≅ ΔBAD

(iv) diagonal AC = diagonal BD

[Hint: Extend AB and draw a line through C parallel to DA intersecting AB produced at E.]



Solution:

To Construct: Draw a line through C parallel to DA intersecting AB produced at E.

(i) CE = AD (Opposite sides of a parallelogram)

AD = BC (Given)

, BC = CE

⇒∠CBE = ∠CEB

also,

∠A+∠CBE = 180° (Angles on the same side of transversal and ∠CBE = ∠CEB)

∠B +∠CBE = 180° (As Linear pair)

⇒∠A = ∠B

(ii) ∠A+∠D = ∠B+∠C = 180° (Angles on the same side of transversal)

⇒∠A+∠D = ∠A+∠C (∠A = ∠B)

⇒∠D = ∠C

(iii) In ΔABC and ΔBAD,

AB = AB (Common)

∠DBA = ∠CBA

AD = BC (Given)

, ΔABC ≅ ΔBAD [SAS congruency]

(iv) Diagonal AC = diagonal BD by CPCT as ΔABC ≅ ΔBAD.